APPLICATION OF STEINER DESIGNS TO FREQUENCY HOPPING SPREAD SPECTRUM SYSTEMS

A Thesis Submitted
in Partial Fulfillment of the Requirement
for the Degree of

MASTER OF TECHNOLOGY

by

SIBAPADA CHAKRABARTI

to the

DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY 2000

CERTIFICATE

It is certified that the work contained in the thesis titled "**Application of Steiner Designs to Frequency Hopping Spread Spectrum Systems**", by **Sibapada Chakrabarti (Roll Number 9810448),** has been carried out under my supervision and this work has not been submitted elsewhere for a degree.

July 3, 2000.

Dr. A. K. Chaturvedi.
Assistant Professor.
Department of Electrical Engineering.
Indian Institute of Technology, Kanpur.

Abstract

A slow frequency hopping code diversity system with a bandwidth efficient modulation scheme based on combinatorial theory of balanced incomplete block (BIB) design — the Steiner design has been proposed. The proposed system uses more than one frequency bins simultaneously to transmit a Steiner symbol. The system performance is evaluated and compared with the M-ary Frequency Shift Keying (MFSK) slow frequency hopping code diversity system for a multi-user environment considering non-fading and frequency non-selective slowly time varying Rayleigh fading channel. Due to the error correcting capability and inherent frequency diversity of the Steiner design, at high SNR the Steiner system shows performance comparable to that of the MFSK system, while requires lesser bandwidth. However, the Steiner system with more number of active elements per block improves the bit error rate performance for a fading channel, but the bandwidth efficiency is reduced. The performance of the system has also been studied with matched frequency hopping, an efficient addressing technique for slow frequency selective dispersive channel

ACKNOWLEDGEMENT

The author would like to express his deep appreciation and gratitude to his thesis supervisor, Dr. A. K. Chaturvedi, for his invaluable guidance and constant encouragement through out the course of this thesis.

The author is thankful to the faculty members of the department of electrical engineering, I.I.T. Kanpur who taught and helped in understanding various courses in the field of communication.

The author is deeply grateful to his close friends Asesh, Kaushik, Satya, Ullash, Bhuwanendra, Anand and Rajinder whose constant support and encouragement have gone a long way towards the successful completion of this work.

The author is deeply indebted to his friends Pradeep, Anil, Satyendra, Pravat and Viswanath for their excellent cooperation and moral support during this work.

Finally, the author wish to record his deep sense of gratitude to his parents, sisters, brothers, sister-in-laws whose constant encouragement and inspiration was behind his education.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Introduction

The potential for communicating with non-fixed points over the horizon without the use of wires was recognized soon after the invention of radio in the late 1800s and its development in the early 1900s. The first major use of such communication was to ships at sea as an aid to navigation and safety. Since those early days, the use of mobile radio has spread dramatically. Today it is used to communicate not only with ships at sea but with land vehicles, aircraft and with people using portable equipment. In the past two decades there has been an increasing interest particularly in land mobile communication for metropolitan and sub-urban areas and mobile satellite communication for rural areas.

In the early mobile communication systems, the fixed base stations were designed to cover as large an area as possible by using maximum affordable power and antennas mounted on high towers. Frequency division was used for spectrum allocation among multiple users. Each base station was assigned a set of disjoint channels. Each frequency was used only once in the service area, leading to high demand for a very limited channel resource.

To improve the spectral efficiency, cellular mobile radio systems have been developed employing the concept of frequency reuse. In a cellular mobile radio system, the service area is divided into smaller geographical areas called *cells*. A single base station is located within each cell and the power radiated by a base station is kept at a level just high enough to cover its corresponding cell. This enables non-adjacent cells to use the same set of frequencies.

As the demand for more user capacity continued to increase, there was a need for an alternative communication technique for a more efficient utilization of the spectrum. In 1970s the spread spectrum (SS) technique was found to be more efficient from the standpoint of spectrum utilization. Since then there has been active research and development of spread spectrum communication systems for commercial applications such as mobile communications, personal communications and wireless LAN.

1.2 Spread Spectrum Communication Systems

Spread spectrum (SS) communication systems were developed to provide secure communications in military environments during World War II. "SS is a means of transmission in which the signal occupies a bandwidth in excess of the minimum necessary to send the information; the band spread is accomplished by means of a code which is independent of the data, and a synchronized reception with the code at the receiver is used for despreading and subsequent data recovery" [1]. The benefits that can be achieved simultaneously by proper spreading the spectrum are

- Anti jamming
- Anti interference
- Low probability of intercept
- Multiple access
- High resolution ranging
- Accurate universal timing

The means by which the spectrum is spread is crucial. Several of the techniques are *direct sequence* (DS), in which a pseudorandomly generated sequence causes phase transitions in the carrier containing data, *frequency hopping* (FH), in which the carrier is caused to shift frequency in a pseudorandom way and *time hopping* (TH), wherein bursts of signal are initiated at pseudorandom times. Hybrid combinations of these techniques have also been proposed.

A fundamental issue in SS is how this technique affords protection against interfering signals with finite power. At the theoretical level SS distributes a relatively low dimensional data signal in a high dimensional environment so that a jammer with a

fixed amount of total power is obliged to either spread that fixed power over all the coordinates, thereby inducing just a little interference in each coordinate, or else place all of the power into a small subspace, leaving the remainder of the space interference free [1].

In addition to its antijam (AJ) capability, an SS signal is generally difficult to detect and even harder to decipher by an unauthorized receiver. This characteristic is usually referred to as a low probability of intercept (LPI). Most interceptors operate as energy detectors, and they have to monitor the received signal long enough to achieve a sufficiently high signal-to-noise ratio (SNR) for reliable detection in the presence of background noise. The LPI advantage of an SS signal is that its power is spread over a bandwidth considerably larger than conventional transmissions, significantly increasing the noise in a receiver that is not privy to the despreading sequence [2].

1.3 Direct Sequence Vs Frequency Hopping

1.3.1 Noise and interference immunity

Frequency hopping spread spectrum (FHSS) systems operate with the SNR of about 18 dB. Direct sequence spread spectrum (DSSS) systems, because of more effective modulation technique used (PSK), can operate with SNR as low as 12 dB [3].

1.3.2 Near - Far problem

The transmitters located close to receivers of other systems generate this problem in DSSS systems and could totally block the activity of the system. On the other hand, if the system is FHSS, the worst case will be that the interferer will block some hops, forcing the system to work in less than optimum conditions, but work!

1.3.3 Multipath effect

Time Delay : In DSSS systems, the chipping process generates a high rate transmitted signal. The symbols of this transmitted signal are much shorter / narrower (in time) than the symbols generated by a FHSS system transmitting the same data rate. Obviously, a narrow pulse (DSSS systems) is more sensitive to time delays than a

wider pulse (FHSS systems) and as a result the FHSS systems have better chances to be undisturbed by the presence of time delays due to multipath effect.

Fading : Due to fading or narrow band jamming if it is desirable to avoid certain regions of the radio frequency band, FHSS system enjoys a distinct advantage over DSSS system.

1.3.4 Throughput

This is the amount of data actually carried by the system (measured in bps). The *rate* of a system is defined as the amount of data carried per second by a system when it is active. DSSS systems are able to transmit data 100% of time, having a high throughput. FHSS systems cannot transmit 100% of the available time (time is lost for hopping and synchronization purposes), having a lower throughput.

1.3.5 Complexity

DSSS radios use PSK modulation, while FHSS radios use FSK modulation (conventionally FH with non-coherent MFSK modulation). PSK implementations are more complex (coherent demodulation, AGC etc.) and therefore require more implementation space.

1.4 Fast vs. Slow Frequency Hopping

In fast frequency hopping (FFH), the hopping speed is greater than the source information rate, and for each bit of information the modulator sends many signals at different frequencies. In slow frequency hopping (SFH), the hopping speed is lower than the source information bit rate (discussed in detail in chapter 2). The main difference is that with SFH we lose blocks of information while with FFH the errors are uncorrelated from one bit to another. Block errors could be a problem for some systems but are acceptable for normal applications. However, it is always possible to repeat useful information in different blocks and obtain a low bit error rate. The main advantage of SFH is that it uses simpler modems than FFH. For example, with typical information bit rate around 16 kbps, FFH would require hop duration of a few μs and this would be expensive to build With the same information bit rate, an SFH modulator

would hop every few ms and the transmission rate would be high [4]. Moreover, it is not difficult to synchronize simultaneous SFH users and this is an advantage for multiple access protocol.

1.5 Motivation for the Present Work

Since the radio spectrum is a limited natural resource, severe congestion of the mobile communication spectrum in some geographical areas dramatically illustrates the need to seek a mobile communication technique that offers the potential for utilizing the spectrum more efficiently. Although different spread spectrum multiple access techniques based on frequencies hopping (slow and fast) have been widely studied and used to fulfill this requirement, there is still scope for these systems to be explored by incorporating code and frequency diversity. This was the motivation for the present work. A slow frequency hopping code diversity system with a bandwidth efficient modulation scheme based on combinatorial theory of balanced incomplete block (BIB) design has been proposed and studied in multi-user environment. Both unfaded and faded channels have been taken into account. The performance of the system has also been studied with matched frequency hopping (MFH), an efficient addressing technique for slow frequency selective dispersive channel

1.6 Organization of the Thesis

The thesis has been organized in five chapters. Chapter 2 describes FFH and SFH systems in detail and existing frequency hopping systems. Chapter 3 introduces a bandwidth efficient modulation scheme adopted and mathematical derivations for bit error rate in unfaded and Rayleigh fading channels. In chapter 4 simulation work has been presented with flow charts, results and discussions. Finally, we come into conclusion in chapter 5 and also suggest some possibilities for further work.

Chapter 2

Frequency Hopping Systems

2.1 Introduction

The classical method of providing multiple access capability is frequency division multiple access (FDMA), in which each user is assigned a particular frequency channel. When all the channels are occupied, the system reaches its capacity and no further user may be accommodated. Another technique for providing multiple access is time division multiple access (TDMA). In TDMA, each user is assigned a particular time slot for transmitting his information. Again, when all time slots are occupied, the system cannot accommodate new users. In contrast to these methods of multiple access, spread spectrum multiple access (SSMA) does not have any sharply defined system capacity. As the number of users increases, the signal-to-interference ratio becomes smaller and there is a gradual degradation in performance until the SNR falls below threshold. Thus the system can tolerate significant amounts of overload if users are willing to tolerate poorer performance.

In a spread spectrum multiple access system each user is assigned a particular code, either a particular PN sequence (DS-SSMA) or a particular frequency hopping pattern (FH-SSMA). FH-SSMA systems are particularly attractive for mobile communication applications because of their frequency diversity that makes them insensitive to frequency selective fading and near-far problem.

2.2 Frequency Hopping Spread Spectrum System

In frequency hopping spread spectrum system, bandwidth is spread by hopping the carrier frequency of the data modulated signal from one frequency to another over a wide band. The specific order in which the frequencies are occupied is a function of a code sequence generally called spreading code or pseudo noise (PN) code. Typically, each carrier frequency is chosen from a set of 2^k frequencies. The spacing between two successive frequencies is approximately equal to the width of the data modulation bandwidth. The transmitted bandwidth is obtained by multiplying the number of frequency slots with the frequency separation between slots. The transmitted spectrum is flat over the band of frequencies.

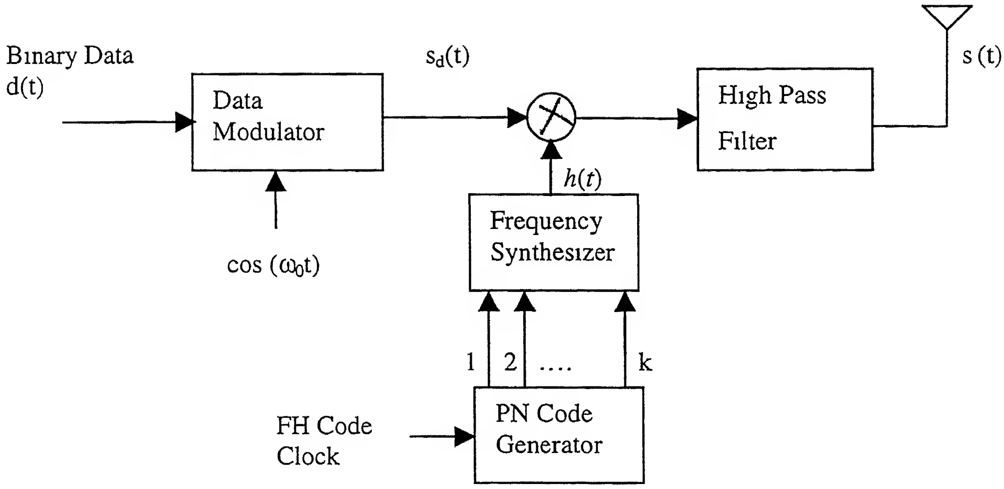
Figure (2.1a) shows a simplified block diagram of an FHSS transmitter. The frequency hopping is accomplished by means of a frequency synthesizer, which is driven by a spreading code or PN code generator. The frequency synthesizer produces one of the 2^k frequencies for each distinct combination of the *k* binary digits of the PN code. The synthesizer output $h(t)$ can be written as [5],

$$h(t) = \sum_{n=-\infty}^{\infty} 2p(t - nT_c)\cos(\omega_n t + \phi_n) \tag{2.1}$$

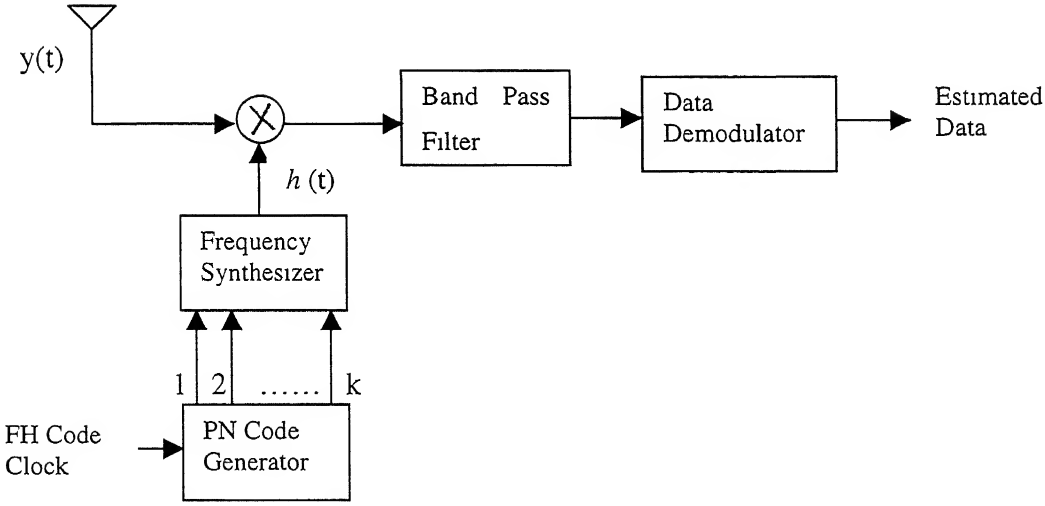
Where $p(t)$ is a unit amplitude pulse of duration T_c starting at time zero and ω_n and ϕ_n are the angular frequency and phase during the *n*th frequency hop interval. The time interval T_c is often called *chip*. The frequency synthesizer output multiplies the data modulated carrier and the high pass filter selects the sum frequency components. Thus the transmitted signal is a data modulated carrier up converted to a new frequency $\omega_0 + \omega_n$ for each chip,

$$s(t) = \left[s_d(t) \sum_{n=-\infty}^{\infty} 2p(t - nT_c)\cos(\omega_n + \phi_n) \right]_{sum\ \ frequency\ \ components} \tag{2.2}$$

If the phase ϕ_n is same, each time $h(t)$ returns to frequency ω_n, that is, if $\phi_n \in \{\phi_m,$ $m = 1,2,\ldots, 2^k\}$, then such a FH system is called coherent FH system. Because of the difficulty of building truly coherent frequency synthesizer most of the FH-SS systems are non-coherent.



(a) Transmitter



(b) Receiver

Figure 2.1: Coherent frequency hop spread spectrum modem [5].

At the receiver the frequency hopping is removed by down converting the received signal with a local oscillator signal which hops synchronously with the received signal. A simplified block diagram of a non-coherent frequency hopping receiver is shown in figure (2.1b) The receiver consists of s frequency synthesizer driven by a PN code generator, a bandpass filter and a data demodulator. The locally generated frequency signal multiplies the incoming signal in a mixer, and the bandpass filter selects the difference frequency component in the mixer output. The bandpass filter output is a normal data modulated carrier and is demodulated by the data demodulator to recover the information.

2.2.1 Noncoherent slow frequency hopping spread spectrum system

Consider a FH system in which data modulation is M-ary FSK. Let the data modulator outputs one of 2^k (k = number of bits) frequencies each kT seconds, where T is the duration of one information bit. Usually the data modulator frequency spacing is kept at least $1/kT$ Hz to ensure orthogonality among transmitted signals. This results in a spectral width of approximately $2^k/kT$ Hz at the data modulator output. After each T_c sec (hop interval), the data modulator output is translated to a new frequency by the frequency hop modulator. When $T_c > kT$ the FH system is called *slow frequency hopping* (SFH) system. Thus in an SFH-SS system the frequency hop rate is less than the information bit rate.

The instantaneous transmitted spectrum of SFH-SS system with $k = 2$ and $n = 3$ is shown as a function of time in figure (2.2a). In this example a new frequency band is selected after each group of 2 symbols or 4 bits and it is then transmitted. In the receiver the incoming signal is down converted by using a local oscillator which outputs the sequence of frequencies 0, $5\omega_d$, $6\omega_d$, $2\omega_d$.... The down converter output is illustrated in figure (2.2b). This signal is demodulated using the conventional methods for non-coherent MFSK.

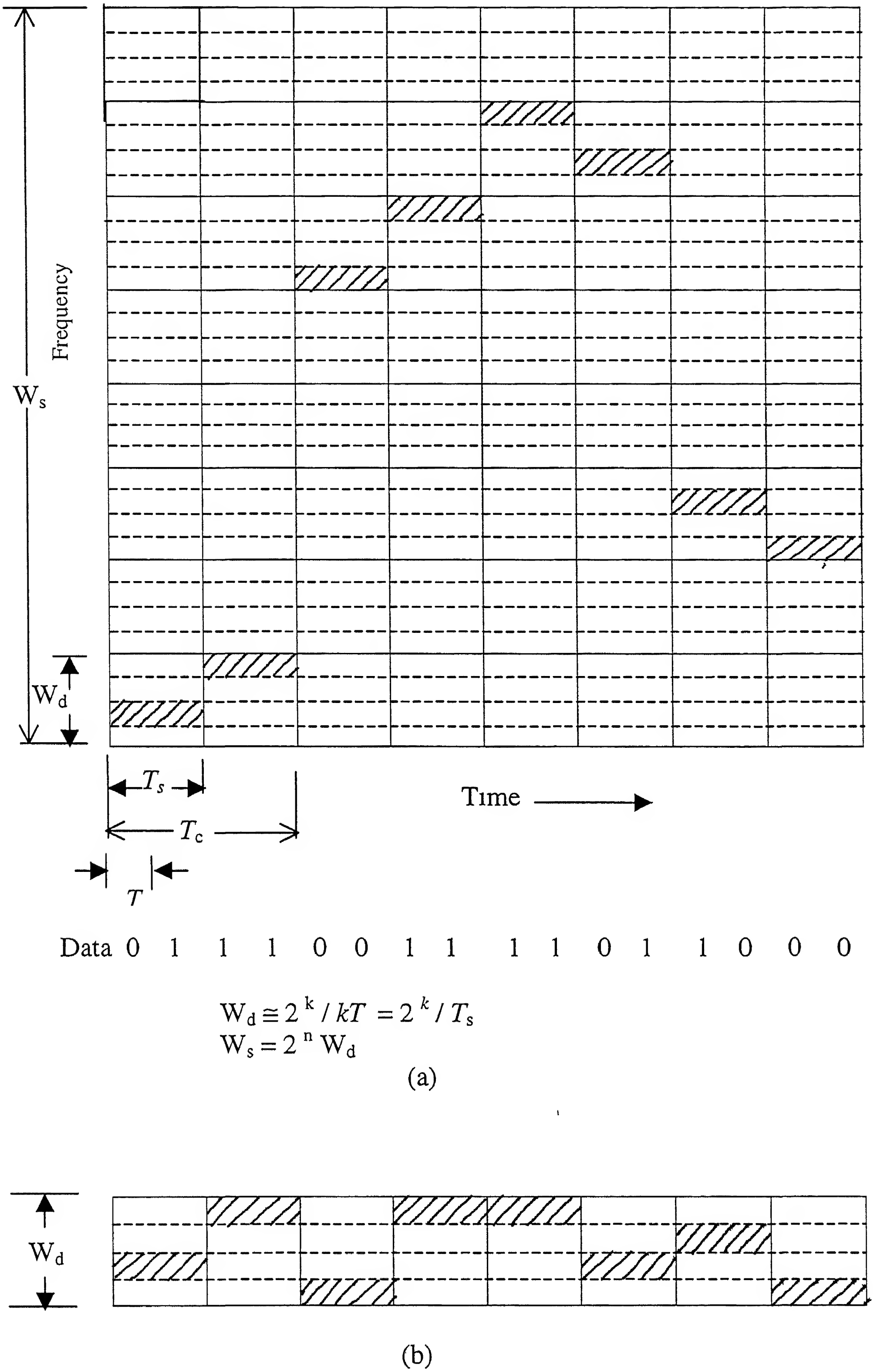
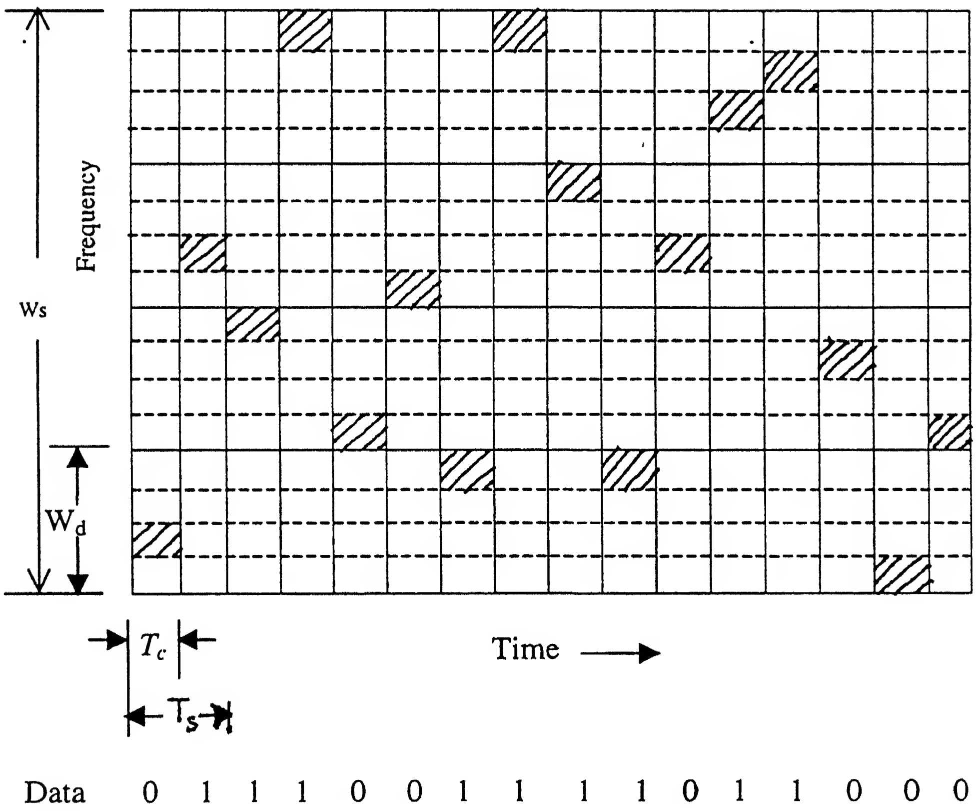


Figure 2.2: Pictorial representation of (a) transmitted signal for an MFSK / SFH-SS system; (b) receiver down converter output [5].

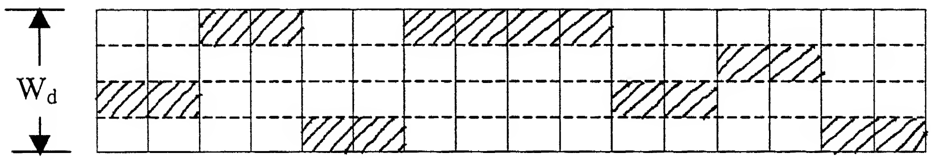
2.2.2 Noncoherent fast frequency hopping spread spectrum system

In contrast to the SFH systems, the hop frequency band changes many times per symbol in a *fast frequency hopping* (FFH) system. For the same MFSK/FH-SS system, which is considered above, the transmitted signal frequency for an FFH-SS system is illustrated in figure (2.3a). The output of the MFSK modulator is one of the 2^k frequencies as before, but now this signal is subdivided into L chips. After each chip, the MFSK modulator output is hopped to a different frequency. Note that as the chip duration or hop interval T_c is shorter than the data modulator output symbol duration T_s, the minimum frequency spacing required for orthogonal signals is now $1/T_c = L/kT$ Hz. The receiver frequency dehopping operation is exactly the same as before. The receiver down converter output is shown in figure (2.3b).



$$W_d \cong 2^k / T_c = L2^k / kT$$
$$W_s = 2^n W_d$$

(a)



(b)

Figure 2.3: Pictorial representation of (a) transmitted signal for an MFSK/FFH-SS system; (b) receiver down converter output [5].

2.3 Existing Frequency Hopping Systems

2.3.1 Cooper and Nettleton DPSK-FHMA system

In 1978, Cooper and Nettleton proposed a SS technique for cellular mobile communications [6], which has been recognized as the first serious approach to applying SSMA methods to the design of a digital cellular system. The proposal involved a frequency hopped multiple access system employing differential phase shift keying (DPSK) as the modulation method. The system achieved greater user density than FM systems.

2.3.2 Bell Labs Multilevel Frequency Shift Keying (MFSK) FFH system

In 1980, Goodman *et al.* proposed a fast frequency hopped SSMA system that employed multilevel (M-ary) frequency shift keying (MFSK) as the modulation method [7]. The proposed receiver, called conventional receiver, estimates solely the transmitted symbol of a particular user following a majority logic decision rule The capacity of this system was shown to be approximately three times the Cooper-Nettleton proposal. The system behaves well in the presence of frequency selective fading thus making it particularly suitable for urban environment. On the other hand, mutual interference among active users limits the spectral efficiency of the system even without any channel impairments.

2.3.3 Multistage decoding of frequency hopped FSK system – Timor

In 1981, Timor [8] considered a technique for improved decoding scheme of FHMA-FSK signals employing Einarsson's construction for frequency hop sequences. He later extended his results by making use of the algebraic structure of the finite field frequency hop sequences for all users. With this improved method, he achieved a better bandwidth efficiency compared to the above systems.

2.3.4 frequency hopped MFSK mobile radio – Haskell

In 1982, Haskell [9] compared the use of random frequency hop sequences (as used by Goodman *et al*), the finite field construction of frequency hop sequences considered by Einarsson, and linearly increasing (chirp) frequency hop sequences via computer simulation. He showed that both the chirp and finite field frequency hop sequence construction yielded improved performance over the random sequence approach for the noiseless, non-fading case.

2.3.5 Slow frequency hopping multiple access (SFHMA) protocols

(a) Orthogonal SFHMA

This is similar to FDMA technique except that instead of fixed frequency channels; we have frequency hopping channels free of interference. The number of channels is same in both the cases. The advantage over FDMA is frequency diversity [4].

(b) Random SFHMA

Most of the existing SFH systems are based on *random* SFH schemes. It does not have reuse clusters and each user has its own pseudorandom sequence and the base stations have very little control over radio spectrum utilization. Though we lose the possibility to optimize cluster size but we have obtained important simplification for network management [4].

(c) Mixed SFHMA

More recent studies focused on another SFH family called *mixed* SFH systems where spectrum utilization is optimized by base station control. In such a system, a predefined

set of hopping sequence is used to ensure mean orthogonality for different cells sharing frequency resources and strict orthogonality within each cell (i.e., no collisions between two distinct channels in the same cell). The SFH900 system is based on such mixed SFHMA protocol combined with time division. This technique combines significant advantages in terms of multipath fading protection, spectrum efficiency and system flexibility in general [10].

2.3.6 Frequency hop codes

The discrimination between the desired signal and the undesired one in the SFH multiple access system may be improved by selecting the *hop code* or *address code* vectors of the users such that there is minimum correlation between them.

In place of the simple random address assignment (RAA) that was used in the original paper [7], Einarsson proposed an address assignment based on the algebra of finite fields to minimize the mutual interference [11]. With Einarsson address assignment (EAA), two transmitted vectors with different hop codes will coincide in, at most, one chip in a synchronous system.

In [9], Haskell investigated another addressing scheme known as chirp address vectors. This scheme assigns to each user vector frequencies, which increase linearly along chips with slope unique to each user. Through computer simulations, Haskell showed that both the chirp and EAA yield improved performance over RAA.

Another class of frequency hop codes used in multi-user communication as well as multi-user radar and sonar systems are based on the concept of congruence equations. This class of codes includes linear congruence codes (LC) [12], cubic congruence codes (CC) [13], hyperbolic congruence codes (HC) [14] and so on. However, hyperbolic codes achieve the minimal probability of error when used for synchronous multiple access spread spectrum communications.

Unfortunately, due to high value of the cross correlation function between different code words in a synchronized environment, HC codes could not support many

system users. Finally, a new type of code called *extended hyperbolic congruential* (HCC) code was introduced in [15] which has lower cross correlation value and can support almost three times as many simultaneous users as the HC code.

2.3.7 Modulation and coding techniques

When Cooper and Nettleton proposed using spread spectrum technique for mobile radio in 1977 [6], they had suggested using DPSK modulation and Hadamard coding for error correction. The reason for using differentially coherent scheme instead of coherent one is that the mobile radio channel introduces random phase shift, and is very difficult to track the phase reference for every frequency channel to which a user may hop.

MFSK was first proposed by Viterbi for airborne mobile users. By using non-coherent detection, this modulation is also immune to the random phase shift due to the channel. To provide frequency diversity, L number of frequencies is used according to the frequency hop code To provide error correction in the MFSK system, the dual-k convolutional code has been suggested. However, assuming Gaussian channel noise and using soft decision receiver, MFSK frequency hopping system showed a drastic increase in the number of users compared to the DPSK system [16].

An extension to the above modulation scheme is a multiple tone FSK system [17], on which the modulator divides its energy among $w > 1$ waveforms. The signaling frames are then represented by arrangements of v binary elements, with w active elements per frame and each arrangement corresponds to an input code word. The major advantage of this scheme resides on the reduction of the number of orthogonal waveforms required in the signaling set to represent a given alphabet and therefore, in a resultant bandwidth efficiency improvement.

2.3.8 Diversity signaling

Frequency hopping systems designed to combat intentional interference have been investigated extensively over the past few years. One particularly effective system that has received much attention is the MFSK frequency hopping system mentioned before.

The performance of these systems in the presence of either tone jamming or partial band noise jamming and either with or without coding has been analyzed and many of the results appear in the recent comprehensive volume by Simon *et al.* [2]. Now, what would be the benefits if the communicator were able to transmit more than one tone on each diversity branch? This part has been analyzed in [18] and it was shown that performance gains could be achieved which, in some cases, were dramatic. A similar technique has been proposed for slow frequency hopping in [19] where at each hop each active transmitter sends its data in $L > 1$ (L is called the diversity of the system) distinct frequency bins to reduce the probability of *confusion* in systems which are subject to jamming. It was found that code diversity scheme of [19] can have a substantially lower symbol error probability than conventional SFH spread spectrum multiple access system in which $L = 1$.

2.4 Matched Frequency Hopping Addressing Scheme

The performance of the high data rate line of sight (LOS) radio communication system for long distance voice and video transmission is limited by the frequency selectivity and channel dispersion causing intersymbol interference (ISI) and correspondingly, transmission errors. For such channels, Rummler [20] has developed a three-path model based on channel measurements performed on typical LOS links in the 6 GHz frequency band. The model is applicable when the delay bandwidth product is less than 1/6. The model uses a channel transfer function given by [20]

$$H(f) = a[1 - \beta e^{-j2\pi(f-f_0)\tau}] \tag{2.3}$$

where a is the overall attenuation parameter, β is called a shape parameter which is due to multipath components, f_0 is the frequency of the fade minimum and τ is the relative time delay between the direct and the multipath components. This simplified model was used to fit data derived from channel measurements.

Matched frequency hopping (MFH) is an efficient signaling technique in frequency selective slowly fading dispersive channel that also enjoys the other properties of spread spectrum system [21]. In the MFH scheme, a suitable channel

measurement technique is assumed to provide the information about the channel transfer function. Using this information, the hopping frequencies are selected to be in those regions of the channel in which signal attenuation is minimum as shown in Fig.2.4. For a delay of $\tau = 3.5$ µs with $f_0 = 326.25$ kHz.

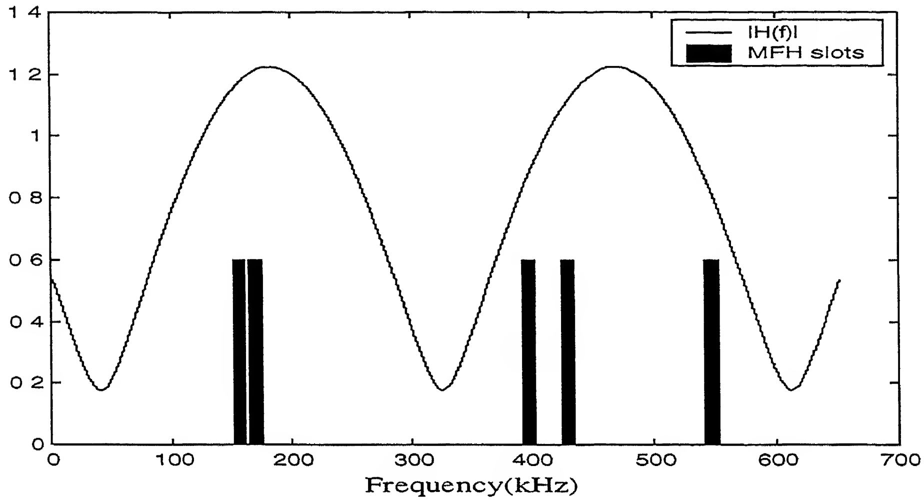


Figure 2 4: Channel transfer function and the choice of frequency hop pattern for $\tau = 3.5$ µs, $f_0 = 326.25$ kHz, $a = 0.7$ and $\beta = 0.75$.

Chapter 3

System Model and Performance Analysis

3.1 Introduction

In a conventional M-ary frequency shift keying (MFSK) system the signaling set is formed by M orthogonal waveforms. The signaling format is represented by frames of M binary elements, with only one active element per frame and each element corresponding to an input code word. An extension to this modulation scheme is a multiple tone FSK (MT-FSK) system, on which the modulator divides its energy among $w > 1$ waveforms The signaling frames are then represented by arrangements of v elements, with w active elements per frame and each arrangement corresponds to an input code word. Here we have adopted a MT-FSK system based on Steiner block design. The major advantage of this scheme resides in the error correcting capability, frequency diversity and the reduction of the number of orthogonal waveforms required in the signaling set to represent a given alphabet and therefore, in a resultant bandwidth efficiency improvement.

These advantages of the Steiner system are combined with a code diversity slow frequency hopping system where each user transmits a given symbol by using more than one simultaneous frequency bins. The system performance is studied for non-fading and frequency non-selective slowly Rayleigh fading channel. For frequency selective slowly fading dispersive channels, an efficient addressing scheme, called matched frequency hopping (MFH), in which the transmitter is capable of adjusting its hopping frequencies in the regions of less channel attenuation, has also been studied.

3.2 Steiner Modulation Scheme

If w active elements are arranged in a frame of v elements, then an alphabet of $\binom{v}{w}$ code words can be represented using a signaling set comprised of only v waveforms. But the major issue is to select only some of the possible arrangements so that the minimum Hamming distance between the frames can be increased. This is because the minimum Hamming distance between the blocks in the signaling set determines the effective diversity of the system [22]. A method has been described in [23], which is based on combinatorial construction called *Balanced Incomplete Block design* (BIB design). The design method is described below

A BIB design is an arrangement of v distinct objects into b blocks such that each block contains exactly w distinct objects, each object occurs in exactly r different blocks, and every pair of distinct objects a_i, a_j occurs together in exactly λ blocks. The sets of all $\binom{v}{w}$ combinations of v objects taken w at a time as blocks would be a complete block design. But a portion of these, in which each pair of objects a_i, a_j occurs the same number of times, is incomplete, but is balanced so far as comparisons between pairs are concerned [23].

So the BIB design satisfies the following conditions:

- Each block contains w elements
- Each element occurs in r blocks and
- Each pair of elements occurs together in λ blocks.

This arrangement of elements is called a BIB design with parameters (v, b, r, w, λ). There are two basic relations among these parameters [23],

$$bw = vr \tag{3.1}$$

$$\lambda(v-1) = r(w-1) \tag{3.2}$$

The first equation counts the total number of single element occurrences and the second equation counts the number of pairs containing a particular element. The number of blocks and the number of repetitions of a particular element are given by

$$b = \lambda \frac{v(v-1)}{w(w-1)} \tag{3.3}$$

$$r = \lambda \frac{v-1}{w-1} \tag{3.4}$$

Here only a particular case, $\lambda = 1$ is considered and the BIB design obtained with this value of λ is known as *Steiner System* and is referred to as $S_w\ (v,\ k)$ where k is the number of input bits represented by the set of blocks. To obtain integer solutions of (3.3) and (3.4), v must satisfy the following condition

$$v \equiv 1\ , \quad \text{or} \quad v \equiv w \bmod w\,(w\text{-}1) \tag{3.5}$$

Therefore only some values of v and w are admissible. The number of elements per block w determines the performance of the system on AWGN channel. Although higher values of w are possible, the bandwidth efficiency of the system is reduced. In short, the parameters of a Steiner system selected to present an M-ary alphabet are such that

- the number of blocks b is sufficiently large to represent each input code word by a block in the Steiner set, and
- the value of v is admissible.

By definition of a Steiner system, where a given pair of elements occur in only one block, therefore for any block B_i, the set of blocks can be partitioned into 3 disjoint subsets,

- B_i
- $X_0\,(B_i)$, containing all orthogonal blocks to B_i
- $X_1\,(B_i)$, containing all non orthogonal blocks in the subsets $X_0\,(B_i)$ and $X_1\,(B_i)$, called block intersection numbers are independent of the selected Block B_i and are given by [24]

$$x_1 = |X_1(B_i)| = w\,(r\text{-}1) = \frac{w}{(w-1)}(v-w) \tag{3.6}$$

$$x_0 = |X(B_i)| = b - x_1 - 1 \tag{3.7}$$

3.3 Bandwidth Efficiency

For a conventional MFSK system, orthogonality in the signaling set is obtained for a frequency separation $\Delta f = 1/T$, with T the symbol interval. Consequently, the bandwidth BW required to represent an M-ary alphabet is given by $BW = (R.M) / k$ where R is the transmission bit rate to transmit k bits of information on the symbol interval T. the bandwidth efficiency R/BW for a conventional MFSK system with diversity L_l, is given by

$$\frac{R}{BW} = \frac{k}{ML_1} \tag{3.8}$$

since the same symbol is repeated L_l times per signaling interval or transmitted over L_l independent subchannels. For Steiner system, only v orthogonal waveforms are required in the signaling set. If the transmission bit rate is the same and each symbol is transmitted in L_2 diversity branches, the bandwidth efficiency is given by

$$\frac{R}{BW} = \frac{k}{vL_2} \tag{3.9}$$

From (3.8a) and (3.8b) it is seen that bandwidth efficiency improvement is related to the ratio ($ML_1/v\,L_2$).

3.4 Description of the System

In the code diversity system a transmitter uses $L > 1$ distinct frequency bins at each hop. L is called the diversity of the system. The available channel bandwidth is divided into q frequency bins, each bin containing v ($< 2^k = M$) frequency slots. Now, each incoming M-ary symbol is modulated (Steiner Modulation) and transmitted in L different frequency slots (one slot each from L different bins), chosen according to the hopping pattern that is known to the receiver. As usual, we assume frequency spacing

and duration that makes the signals orthogonal. The system considered here is a synchronized slow frequency hopping system where hop rate is equal to the symbol transmission rate.

A simple receiver decoding scheme is studied in which, the receiver knows which L of the q frequency bins are being used at any instant, but does not know the frequency slots within the bins and how many transmitters are transmitting a given frequency tone. The transmitter, receiver and the channel model are described in the following sections.

3.4.1 Transmitter block diagram

The frequency hop transmitter based on Steiner multitone FSK system is depicted in figure 3.1. The transmitter selects, according to the input code word (formed by k number of bits), a block $(1, 2, \ldots, w)$ of w elements from $S_w\,(v,k)$ Then a v-ary FSK modulator divides equally its energy into the signals corresponding to the elements forming the selected block, generating a block of orthogonal waveforms $(f_1, f_2, \ldots, f_w)$. Each of these orthogonal waveforms is then frequency hopped according to the hopping patterns generated by a *random* frequency hop generator. The frequency hop generator selects randomly L frequency bins out of q bins available and translates each of the f_i's ($i \in \{1, 2, \ldots, w\}$) to those frequency bins. These $w \times L$ waveforms are then transmitted simultaneously by the transmitter.

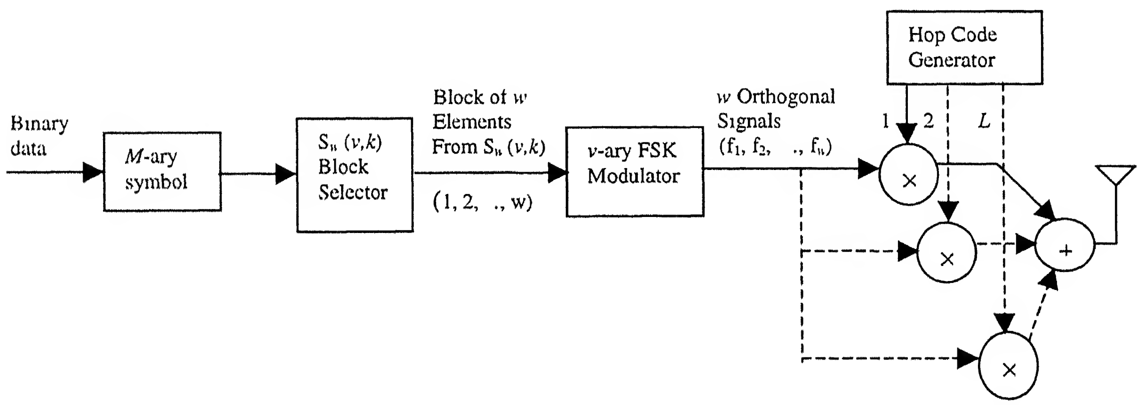


Figure 3.1: Transmitter based on Steiner multitone system.

3.4.2 Receiver block diagram

The receiver consists of a frequency dehopper, band pass filter, a group of v matched filters followed by noncoherent detectors as shown in figure 3.2. The incoming signals are first translated to the base band by multiplying with frequency hop patterns (known as dehopping) and then passing through band pass filter. The translated signals are then detected by v noncoherent detectors (matched filters) in parallel. These v noncoherent detected outputs are combined to obtain a decision variable for each block in the Steiner set. Square-law combining of the outputs (R_{jm}) of the matched filters forms each decision variable, U_j as

$$U_j = \sum_{m=1}^{w} \left| R_{jm} \right|^2 \qquad \text{for } j = 1, 2, \ldots, b \tag{3.10}$$

Finally, the largest decision variable is selected.

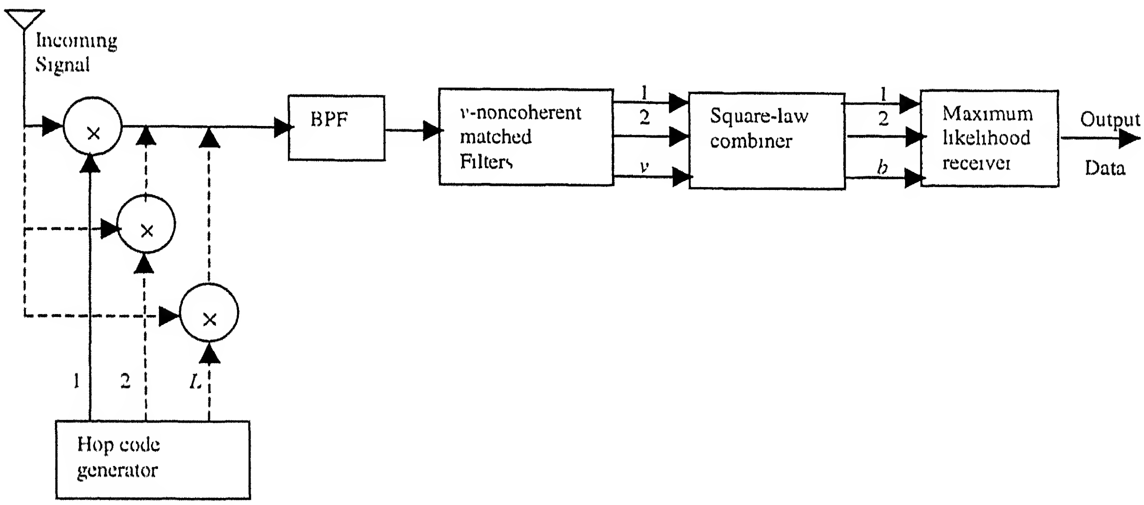


Figure 3.2. Receiver block diagram.

3.4.3 Channel modeling

Because of numerous factors affecting interference in wireless radio networks, certain assumptions are usually made about signal statistics and the propagation model to facilitate the study of the system. The signal passing through the channel is assumed to be affected by multiple access interference (MAI), zero mean additive Gaussian noise. The noise generating from other intentional or unintentional sources is assumed to be Gaussian.

Finally, we consider a channel with randomly time varying impulse responses (e.g ionospheric / tropospheric radio channel) that results in multiple propagation paths. For large number of paths, the received signal can be modeled as a *complex valued Gaussian random process.* The envelope of the received signal at any instant in time is *Rayleigh* distributed and the phase of the signal is uniformly distributed over (0, 2π).

3.5 Performance Analysis of the System

To derive an expression for the bit error rate the following basic assumptions are made.

3.5.1 Basic assumptions

1) The data symbols for each transmitter are statistically independent and take on one of M possible values $\{0, 1, \ldots, M\text{-}1\}$ with equal probability. Each of these M-ary symbols is then Steiner modulated to form a set of w block elements [19].

2) For each hop, each transmitter selects uniformly and independently its L distinct frequency hop bins from the available q bins. Each of $\binom{q}{L}$ possible sets is chosen with equal probability [25].

3) Let the set of N interferers (for a marked transmitter **T**) be partitioned into v symbol groups $G_1, G_2, \ldots, G_v$. Each interferer in group G_i transmits symbol i, $i = 1, 2, \ldots, v$ and we denote the number of such interferers by N_i A *hit* refers to a symbol transmission in a given frequency bin by any two or more transmitters including **T**. A complete hit on symbol is said to occur if all the L bins used by **T** are hit by at least one interferer from symbol group G_s. In the event of a complete hit on L symbols, the receiver takes decision randomly [19].

3.5.2 Analysis

Let $Q\ (i\ /\ n_s)$ be the probability of having exactly i of **T**'s L frequency bins hit by the n_s interferers transmitting symbol 's '. The probability $Q\ (i\ /\ n_s)$ can be calculated using the following recursive equation [19]

$$Q(i/n_s) = \sum_{j=0}^{i} Q(i-j/n_s) \frac{\binom{L-i+j}{j}\binom{q-L+i-j}{L-j}}{\binom{q}{L}} \tag{3.11}$$

with initial conditions

$$Q\ (i\ /\ n_s) = \begin{cases} 1 & for \quad i = 0 \\ 0 & otherwise \end{cases} \tag{3.12}$$

where $\binom{L-i+j}{j}$ denotes the distribution of the hit frequency slots of the marked transmitter and $\binom{q-L+i-j}{L-j}$ denotes the arrangements of non-hit frequency slots of the interfering users.

By symmetry, the symbol error probability is the same regardless of the symbol transmitted by **T**. Assume that **T** transmits a "1". Let P_i, $i \in \{2, 3, \ldots, v\}$ denote the probability that a hit is caused by the n_i interferers transmitting symbol "i". Then

$$P_i = Q\ (1\ /\ n_i), \qquad i \in \{2, 3, \ldots, v\} \tag{3.13}$$

The probability of symbol error due to user interference given the distribution of the N number of interferers can be written as [19]

$$P_e(N, q, L / N_2 = n_2, N_3 = n_3, \ldots\ldots, N_v)$$
$$= \frac{1}{2}\sum_{i=2}^{v} P_i \prod_{j\neq i}(1-P_j) + \frac{2}{3}\sum_{2\le i<j\le v} P_i P_j \prod_{k\neq i,j}(1-P_k) + \ldots\ldots + \frac{(v-1)}{v}\prod_{i=2}^{v} P_i \tag{3.14}$$

where n_2 is the number of users transmitting the symbol "2" and so on. The probability associated with a particular distribution of the interferers is given by the multinomial distribution [26].

$$\Pr\{N_2 = n_2, N_3 = n_3, \ldots\ldots, N_v\} = \frac{N!}{n_2! n_3! \ldots\ldots n_v!}\left(\frac{1}{v}\right)^N \tag{3.15}$$

Further [18]

$$\begin{aligned} P_e(N, q, L) = & \sum_{n_2+n_3+\cdots+n_v=N} \Pr\{N_2 = n_2, N_3 = n_3, \ldots\ldots, N_v = n_v\} \\ & \times P_e(N, q, L / N_2 = n_2, N_3 = n_3, \ldots\ldots, N_v = n_v), \end{aligned} \tag{3.16}$$

Therefore the probability of symbol error due to user interference is obtained using equations (3.11) –(3.16).

In the above analysis we have considered only the user interference. Now we consider the effect of zero mean white Gaussian channel noise for a non-fading channel in the following section.

3.5.3 Non-fading channel

Let $B_j = (b_{j1}, b_{j2}, \ldots, b_{jw})$, for $j = 1, 2, \ldots, b$, be a block in the Steiner set. The receiver is based on the square law combining of the matched filter outputs R_{jm} corresponding to these elements. Therefore, each decision variable is obtained as [17]

$$U_j = \sum_{m=1}^{w} \left|R_{jm}\right|^2 \qquad \text{for } j = 1, 2, \ldots, b \tag{3.17}$$

Let B_i be the transmitted block, with U_i its decision variable and consider a block B_j $(j \neq i)$, with a decision variable U_j. Two cases are then observed [17]:

- $B_j \in X_0(B_i)$, (B_j has no element in common to B_i)
- $B_j \in X_1(B_i)$, (B_j has only one element in common to B_i)

The number of blocks in each case is given by x_0 (equation 3.7) and x_1 (equation 3.6), independent of the transmitted block B_i. This particular intersection property of a Steiner system is used to bound the symbol error probability of the proposed scheme.

Given that B_i is the transmitted block and U_i its decision variable, a decision error is made when a block B_j ($j \neq i$) is detected. Let U_j^0 be the decision variable corresponding to $B_j^0 \in X_0(B_i)$ and U_j^1 is the decision variable corresponding to $B_j^1 \in X_1(B_i)$. The probability of symbol error can be upper bounded by [17]

$$P_d(\gamma, w) \le x_0 P(U_j^o > U_i) + x_1 P(U_j^1 > U_i) \tag{3.18}$$

The first term in (3.18) corresponds to $B_j^0 \in X_0(B_i)$ and therefore no element in common to both decision variables. The decision error probability, $P(U_j^0 > U_i)$ is obtained as

$$P(U_i - U_j^0 < 0) = P(E_0 < 0) \tag{3.19}$$

where the random variable E_0 is defined as

$$E_0 = \sum_{m=1}^{w} (|R_{im}|^2 - |R_{jm}|^2) \tag{3.20}$$

The second term in (3.16) corresponds to $B_j^1 \in X_1(B_i)$ and therefore only one element is common to both decision variables. If $b_{iw} = b_{jw}$ is the intersecting element, then the decision error probability $P(U_j^1 > U_i)$ is obtained as [17]

$$P(U_i - U_j^1 < 0) = P(E_1 < 0) \tag{3.21}$$

where the random variable E_1 is defined as

$$E_1 = \sum_{m=1}^{w-1} (|R_{im}|^2 - |R_{jm}|^2) \tag{3.22}$$

It is shown [27] that the decision error probability for two blocks separated by a Hamming distance of l is bounded by

$$P(\gamma, l) \le \frac{e^{-k\gamma/2}}{2^{2l-1}} \sum_{i=0}^{l-1} \frac{1}{i!} \left(\frac{k\gamma}{2} \right)^i \sum_{s=0}^{l-1-i} \binom{2l-1}{s} \tag{3.23}$$

where γ is the signal-to-noise ratio per bit. Therefore, equation (3.19) and (3.21) can be obtained as [17]

$$P(E_0 < 0) = P(\gamma, l = w), \tag{3.24a}$$

$$P(E_1 < 0) = P(\gamma, l = w-1) \tag{3.24b}$$

Let P_u = Pr {simultaneous user interference on $i \leq w$ Steiner elements}

$$= \binom{w}{i} P_e^{\,i}(1 - P_e)^{w-i} \tag{3.25}$$

where P_e is the probability of user interference in a Steiner symbol given by (3.16).

For a Steiner set it has been shown that two blocks intersect at most on a single element, then its minimum Hamming distance $d_{\min}$ is $(w$-1). If $(w$-1) slots are hit, then any block in X_1 (B_i) (besides B_i), constitutes a possible block to be detected, while if all w frequency slots are hit, any block in the Steiner system is a possible block. In both cases the receiver selects the largest decision variables formed with the metric given by (3.17). If $P_d(\gamma, w|j)$ is the probability that j slots (among w frequency slots) in a Steiner block are hit by other interfering users, the probability of Steiner symbol error when $(w$-1) slots are interfered is given by [17]

$$P_d(\gamma, w|w-1) \leq x_1 P(E_1 < 0) \tag{3.26}$$

When all w transmitted slots are hit, the Steiner symbol error probability is bounded by [17]

$$P_d(\gamma, w|w) \leq x_0 P(E_0 < 0) + x_1 P(E_1 < 0) \tag{3.27}$$

Where E_0 and E_1 are calculated using (3.24a) & (3.24b) respectively. Therefore, the Steiner symbol error probability is given by

$$P_{st} = \sum_{i=d_{\min}/2}^{w} P_u P_d(\gamma, w|i) \tag{3.28}$$

Since each Steiner symbol was transmitted by L simultaneous frequency slots according to the generated hop code, we have L detected Steiner coded symbols at the end of the detection process. If a majority decision rule is used, the output symbol error probability is obtained as

$$P_{sy} = \sum_{J=(L+1)/2}^{L} \binom{L}{J} P_{st}^{\,J} (1 - P_{st})^{L-J} \tag{3.29}$$

The equivalent probability of a binary digit error is given by [27]

$$P_b = \frac{2^{k-1}}{2^k - 1} P_{sy} \tag{3.30}$$

3.5.4 Rayleigh fading channel

In this section we derive the bit error rate performance when the signals are transmitted over a frequency non-selective slowly fading channel This type of channel results in multiplicative distortion of the transmitted signal. The received equivalent low pass signal in one signaling interval is represented by [27]

$$r_\kappa(t) = \alpha_\kappa e^{-j\phi_k} u_m^k(t) + z_\kappa(t), \qquad 0 \le t \le T \tag{3 31}$$

Where $\kappa = \{1, 2, \ldots, w\}$, $m = \{1, 2, \ldots, v\}$, $\{u_m^\kappa(t)\}$ are the equivalent low pass transmitted signals and $z_k(t)$ are mutually statistically independent and identically distributed Gaussian noise random processes.

The probability density function of U_1 (the decision variable containing the Steiner symbol) is given by [27]

$$p(U_1) = \frac{1}{(2\sigma_1^2)^w (w-1)!} U_1^{w-1} \exp(-\frac{U_1}{2\sigma_1^2}) \tag{3 32}$$

where $$\sigma_1^2 = \frac{1}{2} E(|R_{1m} + Z_{1m}|^2) = 2\xi N_0 (1 + \gamma_c) \tag{3.33}$$

and Z_{1m} is the received noise component, ξ is the signal energy per bit and γ_c is the average SNR per branch given by

$$\gamma_c = \frac{k\xi}{wN_0} \sum_{m=1}^{w} E(\alpha_m^2) = \frac{k}{w} \gamma_b \tag{3.34}$$

where γ_b is the average SNR per bit.

On the other hand, the probability distribution of U_j (the decision variables not containing the Steiner symbol, where $j = 2, \ldots, M$) is given by

$$p(U_j) = \frac{2}{(2\sigma_j^2)^w (w-1)!} U_j^{w-1} \exp(-\frac{U_j}{2\sigma_j^2}) \tag{3.35}$$

where $$\sigma_j^2 = 2\xi N_0 \tag{3.36}$$

Now the probability that $U_1 < U_2$ is given by [27]

$$p(U_1 < U_2) = 1 - \int_0^\infty \left[\left(1 - \exp(\frac{U_1}{2\sigma_2^2}) \right) \times \left(\sum_{p=0}^{w-1} \frac{1}{p!} (\frac{U_1}{2\sigma_2^2})^p \right) \right] \times \frac{1}{(2\sigma_1^2)^w (w-1)!} U_1^{w-1} \exp(-\frac{U_1}{2\sigma_1^2}) dU_1 \tag{3.37}$$

The closed form solution of this equation is given by [27]

$$p(U_1 < U_2) = \left(\frac{1}{1+\gamma_c} \right)^w \sum_{i=0}^{w-1} \binom{w-1+i}{i} \times \left(\frac{1+\gamma_c}{2+\gamma_c} \right)^i \tag{3.38}$$

Using the union bound

$$\Pr\left(\bigcup_{i=1}^{N} A_i \right) \le \sum_{i=1}^{N} A_i \tag{3.39}$$

the detection error probability can be upper bounded by

$$P_d(\gamma_c, w) \le (M-1) p(U_1 < U_2) \tag{3.40}$$

This result is used (with $\gamma = \gamma_c$) in (3.24a) with w and in (3.24b) with $w = w - 1$ to obtain the average decision error probability. The resultant equations are replaced in (3.27) to bound Steiner symbol error probability. Finally, the expression for bit error rate is obtained by using the equations (3.28) – (3.30).

3.6 Channel Matched Frequency Hopping Patterns

The frequency hop (FH) patterns we have considered so far were uniformly distributed random numbers. Now we will select the frequency patterns matched to the frequency selective channel characteristics (attenuation and dispersion). This selection is such that the used frequency slots are concentrated in the frequency subbands, which are characterized by high channel transmission coefficients. Mathematically, this is equivalent to selecting a nonuniform FH pattern such that its probability density function (pdf) $p_F(f)$ in the available channel bandwidth is proportional to the channel transmission coefficient $|H(f)|^2$, i.e.[21],

$$p_F(f) = \begin{cases} \eta |H(f)|^2 & f \in W \\ 0 & otherwise \end{cases} \tag{3.41}$$

where the constant η is selected such that the integration of $p_F(f)$ over the available channel bandwidth W is equal to unity This assures that the selected frequencies will be concentrated in frequency regions with minimum attenuation.

After the specifying the pdf of the FH pattern, the task now is to solve the inverse problem, namely, how to determine the channel matched FH pattern. If one start with uniformly distributed FH pattern in which the frequencies can be treated as uniformly distributed random variable, denoted by X in the domain (0, 1), we look for a transformation that yields another random variable with the pdf function given by (3.45). It can be shown that this transformation is given by [28]

$$F_j = Q(x_j) \tag{3.42}$$

where $Q(x_j)$ is the inverse transformation

$$Q(x_j) = P_F^{-1}(x_j) \tag{3.43}$$

and

$$P_F(x) = \int_{f_{min}}^{x} p_F(f)df = \eta \int_{f_{min}}^{x} |H(f)|^2 df \tag{3.44}$$

where $P_F(x)$ is the cumulative density function (CDF) of F and $f_{\min}$ is the lower frequency edge of the channel bandwidth, x_j is a sample value of the uniformly distributed random variable X with $0 \leq x_j \leq 1$.

Chapter 4

Simulation Models and Results

4.1 Introduction

The error probability for a single communication link has been studied considering the effect of user interference and channel characteristics. This chapter describes the simulator models and the choice of various simulation parameters to evaluate the system performances. The Gaussian noise is added at the front end of the receiver. The symbols transmitted by the users are assumed to be aligned in time. The simulations were carried out by using MATLAB.

4.2 Simulation Parameters

The simulation parameters are chosen depending on the constraints of computation speed and the resources available. The values of the parameters taken for simulation are given in table 4.1 & 4.2. Choices of various simulation parameters are described as follows.

4.2.1 Choice of frequency hop bands

The incoming bit error rate is assumed to be 1 kbps and the frequency hop band starts from 1.25 kHz and is extended up to 651.25 kHz containing a total of 100 hop bands or frequency bins each of width 6.5 kHz as shown in figure 4.1. Since the bit rate is 1 kbps, the symbol rate is 250 sy/sec (M-ary FSK with $M = 16 = 2^4$) and the symbol duration (T_s) is 4 ms. In order to maintain orthogonality between frequency slots the frequency separation is kept as 500 Hz (multiple of $1/T_s$.). Each frequency hop bin is 6.5 kHz wide and consists of 13 slots each of width 500 Hz. Therefore, the frequency of i th slot within the n th frequency bin is given by

$$f_{ni} = f_1 + (i-1)\times 500 + n\times 6500 \quad Hz \tag{4.1}$$

where f_1 = 1.5 kHz, i = {1, 2, ... , 13} and n = {0, 1, ... , 99}. The value of i is determined by the arrangements of k input bits and the design of the Steiner blocks, n is determined by the generated hop code. Since the maximum possible signal frequency used is 651.25 kHz, the sampling rate was taken to be 4 times of this, i.e.2.605×10^6 samples/sec.

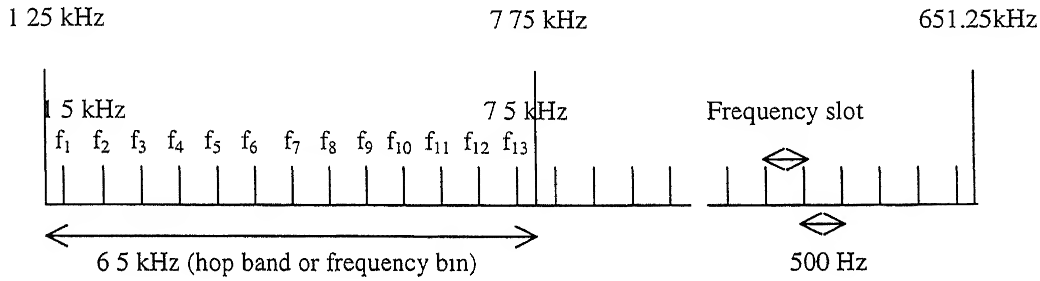


Figure 4.1. Frequency allocation for the Steiner system.

4.2.2 Transmitter

The transmitter block diagram is shown previously in figure 3.1 (chapter3). The block diagram of the uplink simulator is shown in figure 4.2. Multiple user interference is implemented by generating the signals for a number of interfering mobile users within the system. Each interfering user has its own address code and data bits as the desired one. The user bits as well as symbols are aligned in time scale. The transmitted symbols for different users arrive at the base station via different multipath channels and constitute multiple access interference (MAI). In the simulator for Gaussian only (non-fading) noise channel, we assume that we have perfect power control i.e. all the users transmit at equal power level and the received signals at the base station for all the users are at the same average power level. But the situation is quite different for frequency non-selective slow Rayleigh fading channel where the received signal amplitude is Rayleigh distributed and the phase is uniformly distributed.

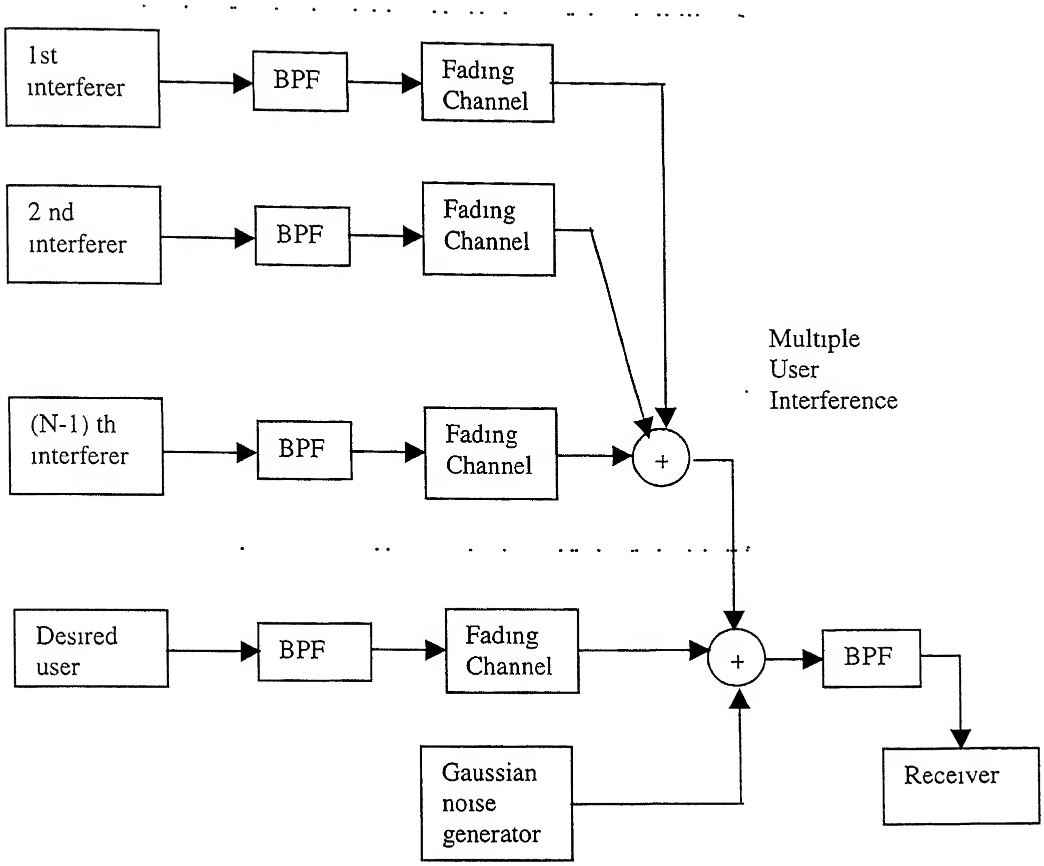


Figure 4.2. Transmitter block diagram for N number of users.

4.2.3 Receiver

The structure of the non-coherent detectors and the square-law combiner is shown in figure 4.3. Length of the hop code is L and we have shown only one branch of the receiver block diagram (Fig. 3 2). In some channels the time variations may be sufficiently fast to preclude the implementation of coherent detection and the signal phase cannot be estimated perfectly. That's why we have used non-coherent detection. Since each Steiner block is transmitted in w different frequencies as well as in w different time slots, square-law combining technique, as an alternative to envelope detector, is used at the receiver for the present case because the former is much easier to analyze.

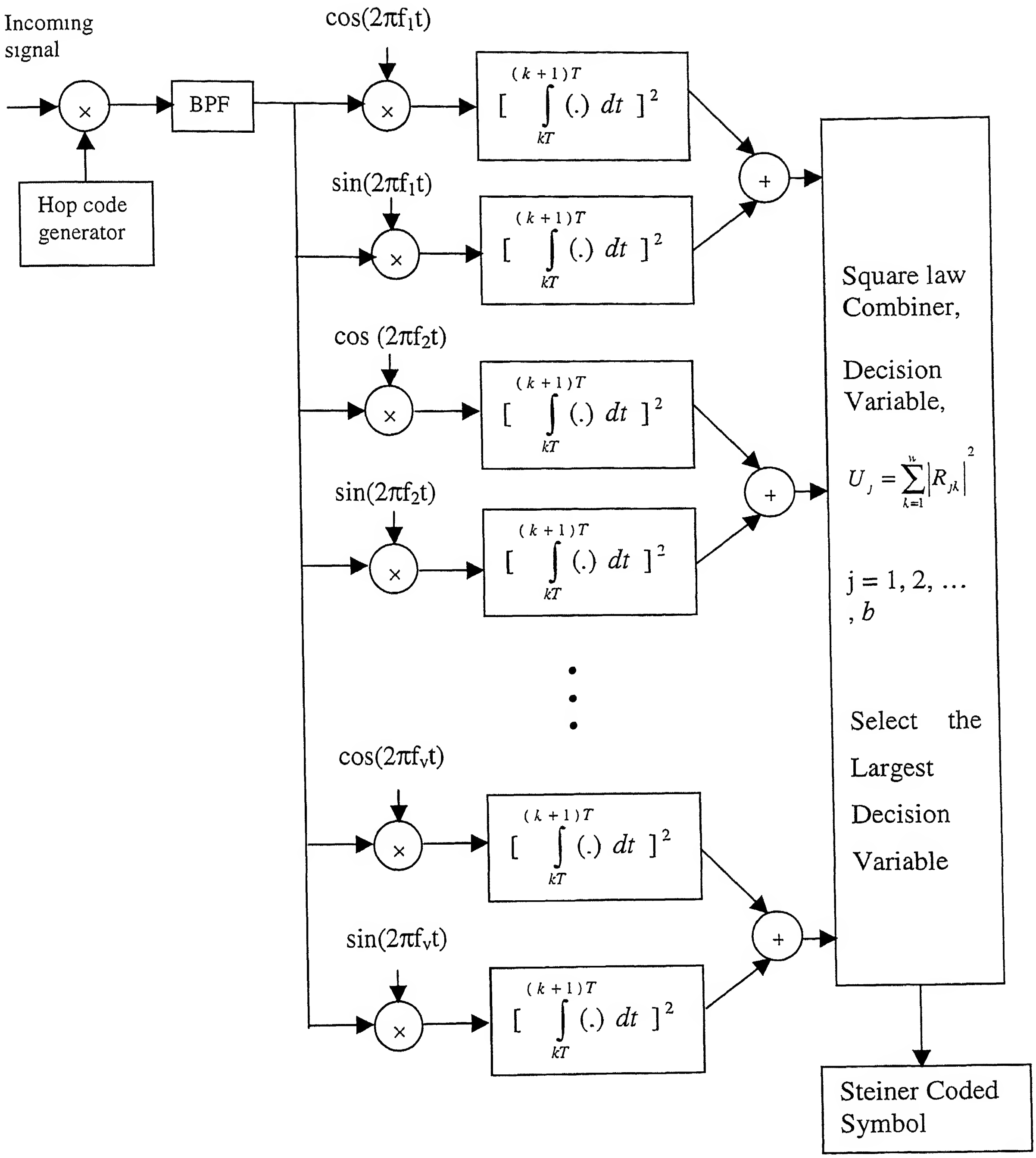


Figure 4.3: ν Non-coherent detectors and the combiner block diagram

4.2.4 Steiner Block design

Steiner blocks are designed with help of incidence matrix. This is a matrix $A = (a_{ij})$, $i = 1, 2, \ldots, v$, $j = 1, 2, \ldots, b$, where if $a_1, \ldots, a_v$ are the objects and $B_1, \ldots, B_b$ are the blocks, we have

$$a_{ij} = \begin{cases} 1 & \text{if} \quad a_i \in B_j \\ 0 & \text{if} \quad a_i \notin B_j \end{cases} \tag{4.2}$$

The basic requirements for the block design are expressed by the matrix equations

$$AA^T = B = \begin{bmatrix} r & \lambda & \dots & \lambda \\ \lambda & r & . & . \\ . & & & . \\ . & . & . & \\ . & & & . \\ \lambda & & . & r \end{bmatrix} = (r - \lambda)I_v + \lambda J_v \tag{4.3a}$$

$$w_v A = k w_b \tag{4.3b}$$

Here, J_v is the $v \times v$ matrix of all 1's and w_v and w_b are respectively the vectors of v and b 1's. The off diagonal elements of B counts the occurrences of the pairs a_i, a_j and so is λ in every instance. In our case the values of different parameters for block design (obtained with the help of the equations (3.1) – (3.5)) are as follows.

$$\lambda = 1, w = 3, v = 13, b = 26, r = 6 \tag{4.4}$$

The possible blocks are given below of which the blocks under the dotted boundary are used in our system.

1 2 3,	2 4 6,	4 3 8,	7 8 13,	3 5 12
1 4 5,	2 5 7,	4 7 9,	7 10 12,	3 6 9
1 6 7,	2 8 10,	4 10 13,	8 5 11,	3 9 13
1 8 9,	2 9 12,	4 11 12,	8 6 12,	5 6 10
1 10 11,	2 11 13,	7 3 11,	6 9 11,	5 10 13
1 12 13,				

4.2.5 Gaussian noise simulator

The additive white Gaussian noise (AWGN) added at the front end of the receiver is generated by Gaussian random number generator function 'randn' (MATLAB). The

variance of the noise distribution depends on the Signal to Noise ratio (SNR) or E_b/N_0 at the receiver front end. The power spectral density of the Gaussian noise signal after passing through the band pass filter of frequency band (1.25 to 651.25) kHz is shown in figure 4.4. The band pass filters are designed with the help of two MATLAB functions namely 'fir1' and 'filtfilt'.

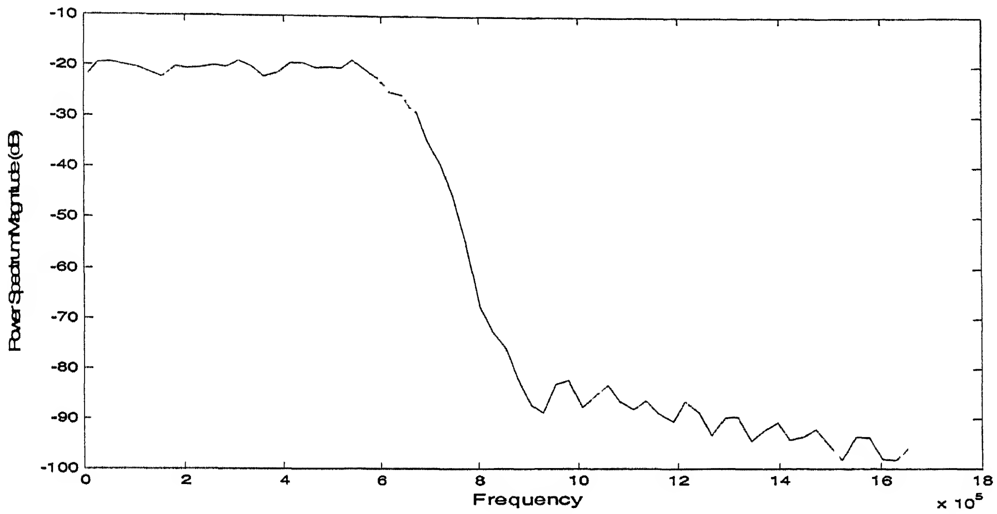


Figure 4.4:Power spectral density (PSD) of the band pass (1.25kHz to 651.25 kHz) white noise.

4.2.6 Rayleigh fading channel simulator

The Rayleigh variable α_k (equation 3.31) is generated as

$$\alpha_k = \sqrt{z_1^{\,2} + z_2^{\,2}} \tag{4.2}$$

where z_1 and z_2 are zero mean statistically independent Gaussian random variables each having a variance σ^2. We consider the power normalized to one as

$$E\{\alpha_k^{\,2}\} = 2\sigma^2 = 1 \tag{4.3}$$

which gives a variance of 0.5 for each of the Gaussian variables. The Rayleigh fading produces multiplicative noise with the signal. The signal phase is uniformly distributed over the interval $(0, 2\pi)$.

4.2.7 Generation of random address codes

The hop code of length L for each user is generated by using the MATLAB function 'randint', which produces uniformly distributed random number between a specified range. The numbers were generated within the range from 0 to 99 because $q = 100$ frequency bins are available for hopping. At each hop we are generating L uniformly distributed random numbers that corresponds to L frequency bins each of width 6.5 kHz (Fig. 4.1). Now, a given Steiner block of w elements is translated or frequency shifted to these L frequency hop bins and then transmitted simultaneously. The process is repeated for other Steiner blocks. The random address code models are often used in an attempt to match certain characteristics of extremely complex hopping patterns, which have very long periods. Also, random process models serve as substitutes for deterministic models when the communications engineer is given little or no information about the structure of the hopping patterns to be used in the system.

4.2.8 Simulation Parameters for Steiner system

Table 4.1: Simulation Parameters for Steiner system

Simulation Parameters	Values
1. Input bit rate	1 kbps
2. Symbol rate	250 symbols/s
3. Symbol duration	4 ms
4. Sampling rate	2.605×10^{6} samples/s
5. Channel bandwidth (W)	650 kHz
6. Number of frequency bins (q)	100
7. Hop bandwidth (frequency bin width)	6.5 kHz
8. Number of slots per hop band (v)	13
9. Width of each frequency slot in a frequency bin	500 Hz
10. Code diversity (L)	3
11. Total number of bits transmitted for each data	5000-10000

12. Number of input bits (k) taken at a time	4
13. Parameters for Steiner design : 1) b 2) v 3) w 4) λ 5) r	 26 13 3 1 6
14. Matched frequency hopping (MFH) parameters 1) Time delay (τ) 2) f_0 3) a 4) β	 6 µs 326.25 kHz -3 dB (or 0.7) 0.75

4.2.9 Simulation parameters for MFSK slow frequency hopping system

The simulation parameters, which are used particularly for the MFSK slow frequency hopping code diversity system, are given in table 4.2. Other parameter values are same as the Steiner system.

Table 4.2: Simulation Parameters for MFSK system

Simulation Parameters	Values
1. Sampling rate	3.205×10^6 samples/s
2. Channel bandwidth (W)	800 kHz
3. Hop bandwidth (frequency bin width)	8 kHz
4. Number of slots per hop band	16
5. Code diversity (L)	5

4.3 Simulation Flow Charts

In the simulation flow charts we have considered randomly generated (uniformly distributed) hop codes only. For matched frequency hopping codes we will follow the

procedure given in section 3.6 (chapter 3) and this procedure is graphically represented by Fig. 4.8.

4.3.1 TRANSMITTER

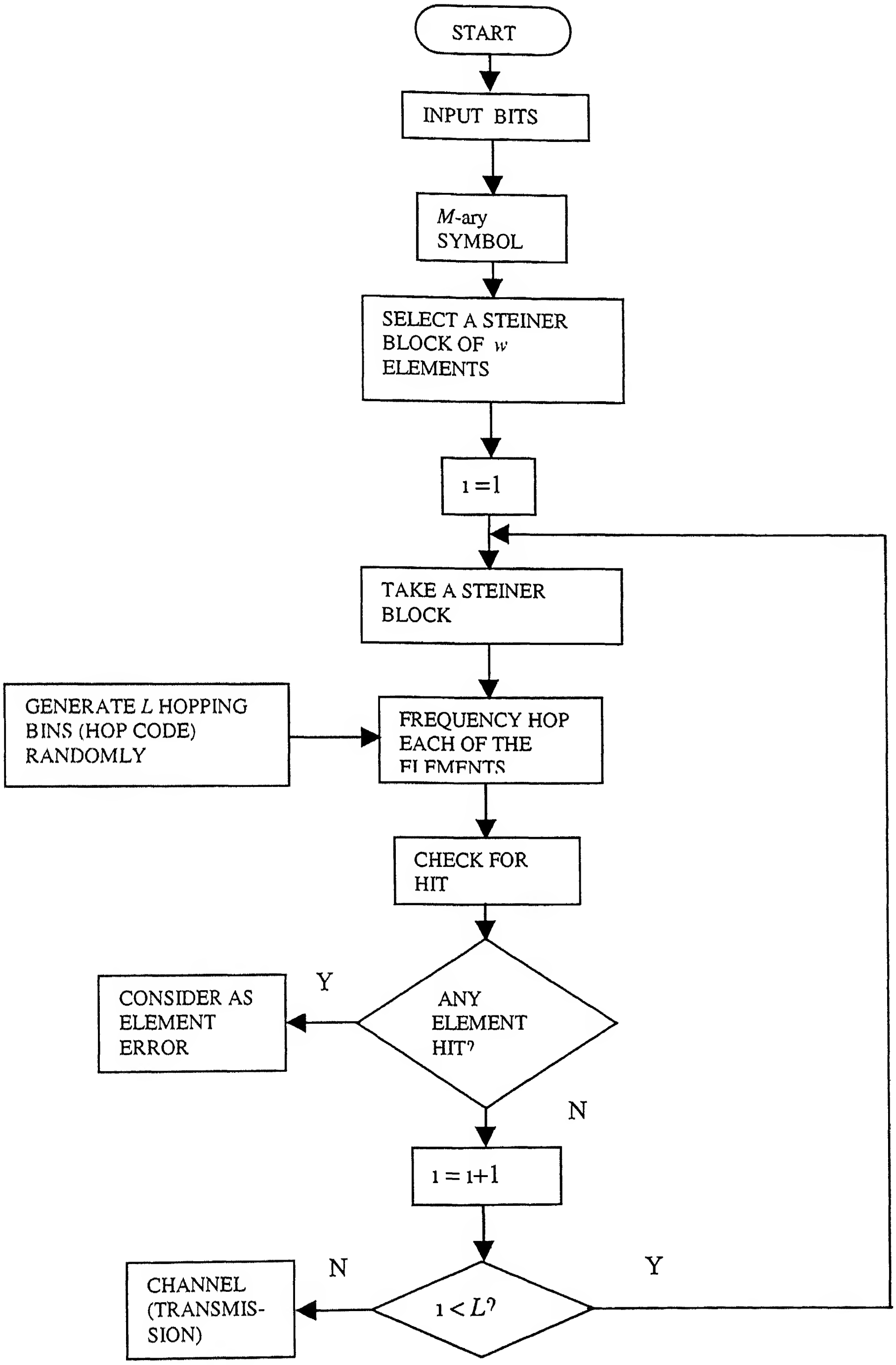


Figure 4.5: Simulation flow chart for the transmitter.

4.3.2 CHANNEL

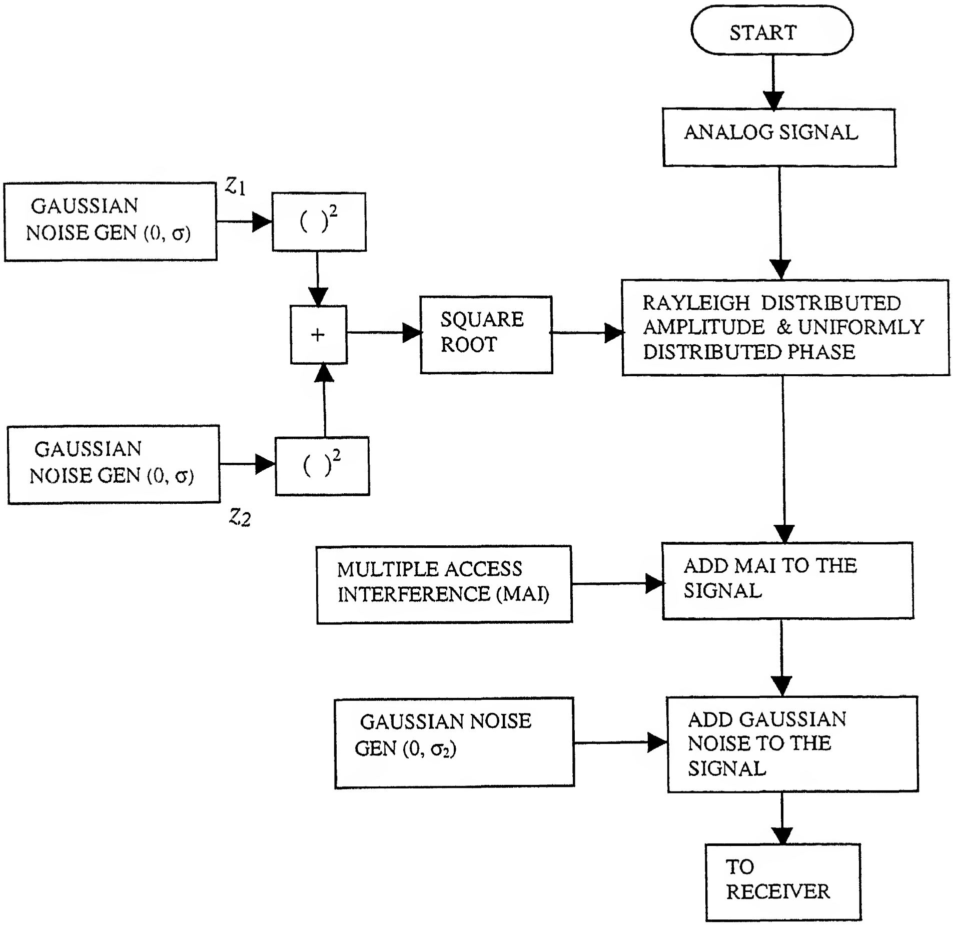


Figure 4.6: Simulation flow chart for the frequency non-selective Rayleigh fading channel with MAI and zero mean additive Gaussian noise.

4.3.3 RECEIVER

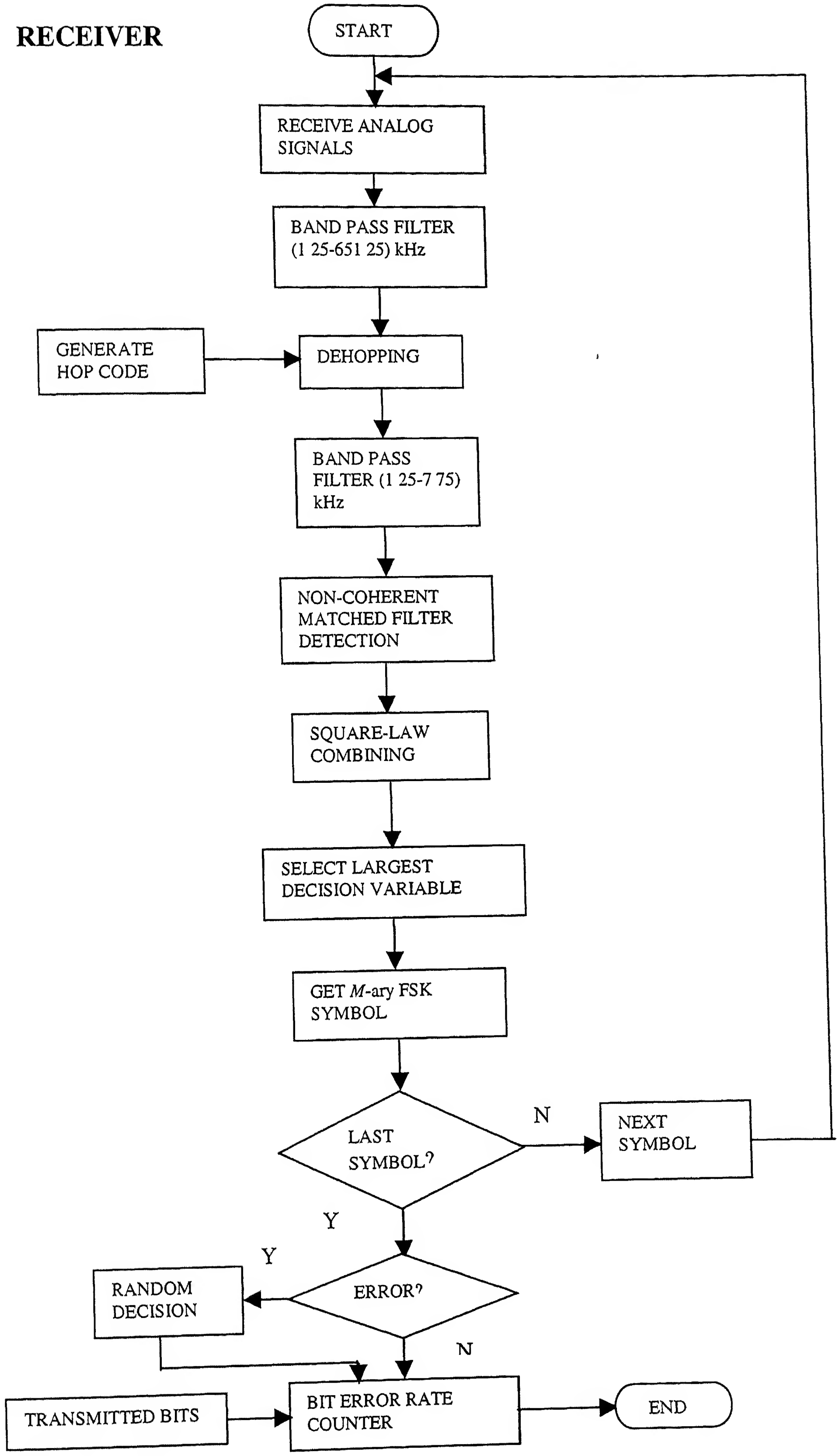


Figure 4.7: Simulation flow chart for the receiver.

4.4 Matched FH Pattern Generation

The generation of matched frequency hopping patterns is graphically presented in Fig. 4.8 for a delay of $\tau = 6\ \mu s$ with $f_0 = 326.25$ kHz, a = -3dB (0.7) and $\beta = 0.75$ (the symbols have been defined in section 3.6 of chapter 3).

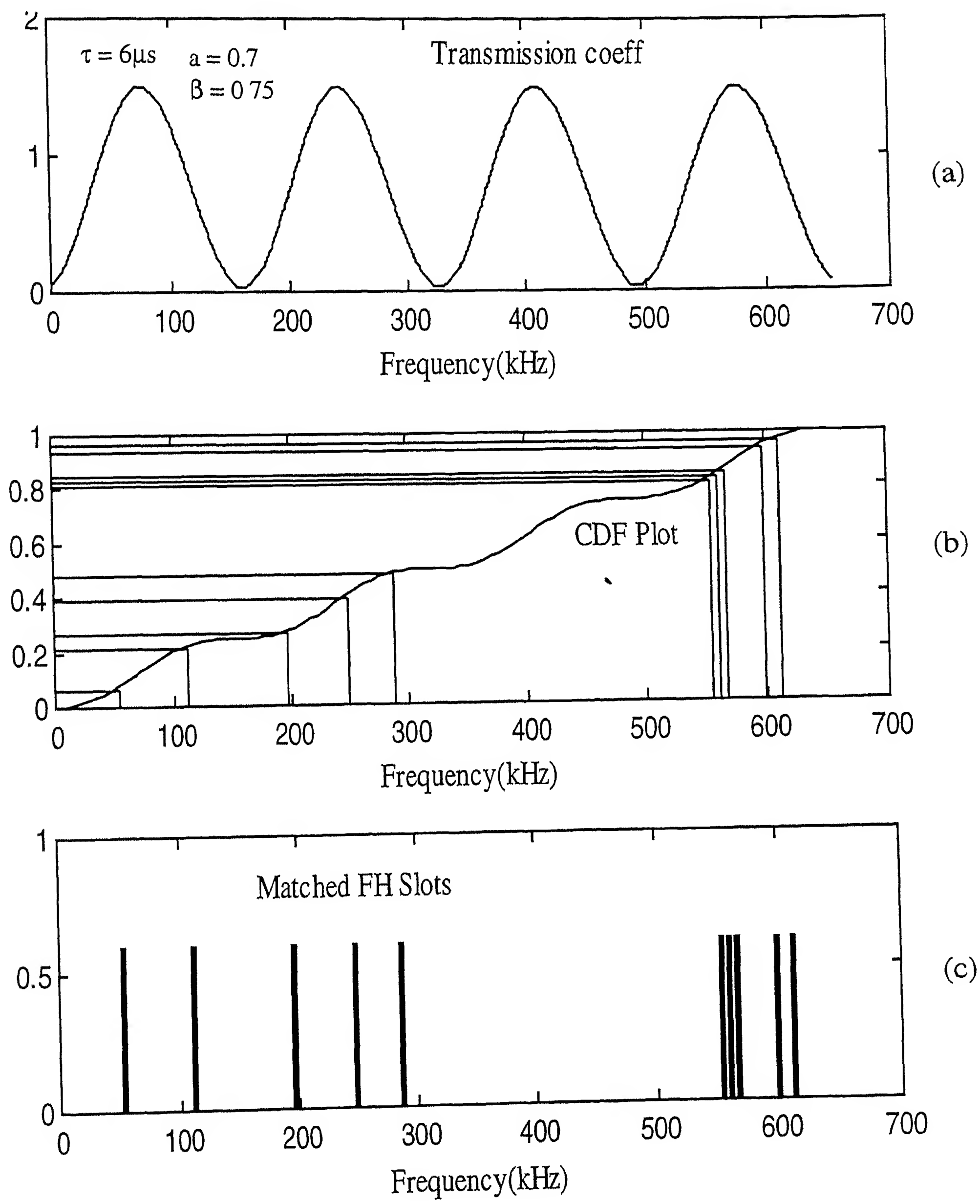


Figure 4.8: A graphical representation of the procedure used to find the matched FH patterns. (a) Channel transmission coefficient. (b) CDF of the MFH frequencies and uniformly distributed FH pattern. (c) The matched FH patterns.

The value of the constant η that was used to make the integration of equation 3.44 equal to unity over the available channel bandwidth was found to be 5.098×10^5.

4.5 Theoretical and Simulation Results for non-fading Channel

4.5.1 Choice of optimum diversity

The modulation system considered here is Steiner triple system, i.e. S_3 (v, k) where v = 13 and k = 4 (number of input bits taken together to form an M-ary block). The simulation plot of bit error rate (BER) vs. diversity (L) for different number of user (N) for both Steiner and MFSK-SFH systems at Eb/No of 25 dB with q = 100 frequency bins, M = 16 and the number of Steiner elements v = 13 is shown in Fig. 4.9. The dotted curves show the performance for MFSK-SFH system. For diversity less than 3, Steiner system performance is much better than that of MFSK-SFH system, but after that the MFSK system performance improves and at L = 5 becomes comparable even better than that of the Steiner system. The optimum diversity for Steiner system is taken as 3 whereas the diversity is 5 for MFSK system

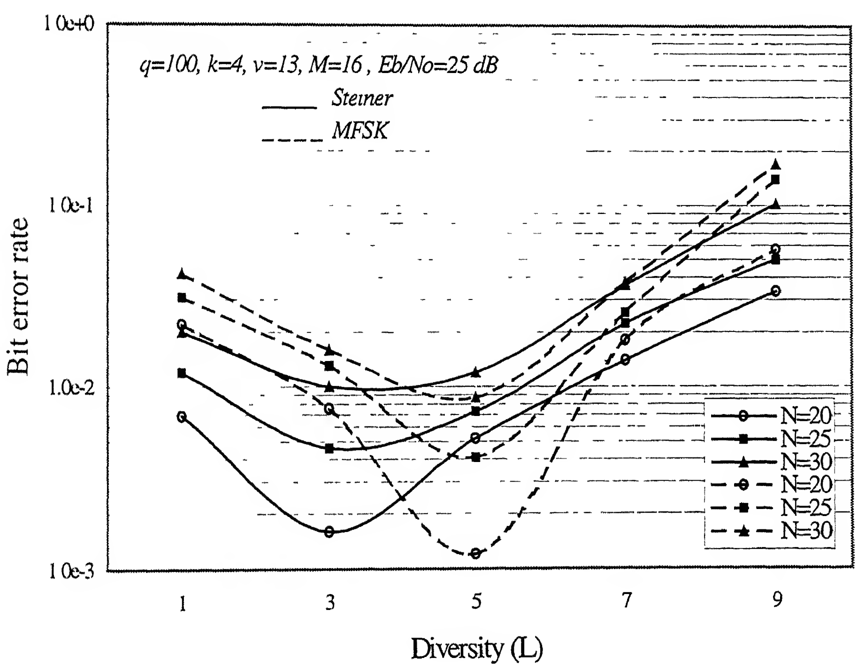


Figure 4.9: Bit error rate vs. diversity (L) plot for different number of user (N) at Eb/No = 25 dB, for non-fading channel.

4.5.2 Theoretical results

Figure 4.10 shows the theoretical plot (equation 3.30) of upper bounds for bit error rate against Eb/No (dB) with number of user (N) as a parameter. The parameters (*q, M, k, L, v*) have the same values as mentioned above. The curves are plotted for 20, 25 and 30 users. At any value of SNR (Eb/No), bit error rate increases with the number of users. For lower values of SNR both Gaussian channel noise and multiple access interference (MAI) are present, but at the higher SNR MAI is dominating and BER is practically independent of SNR.

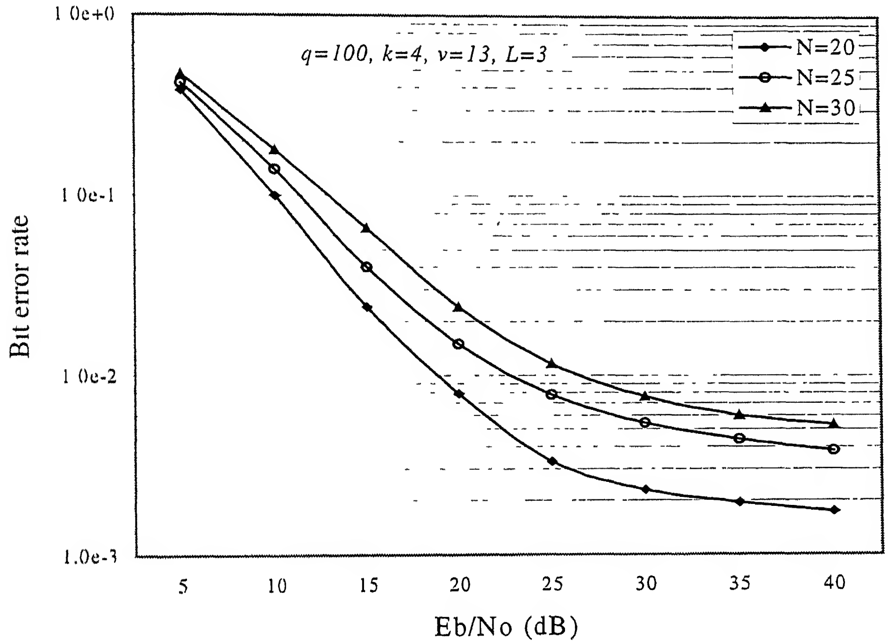


Figure 4.10: Theoretical plot of upper bounds for Steiner system with N as parameter for non-fading channel.

In Fig 4.11, theoretical upper bound (equation 3.10) for bit error rate is plotted for Steiner system against number of user (N) with Eb/No (dB) as a parameter. The parameters are same as mentioned above.

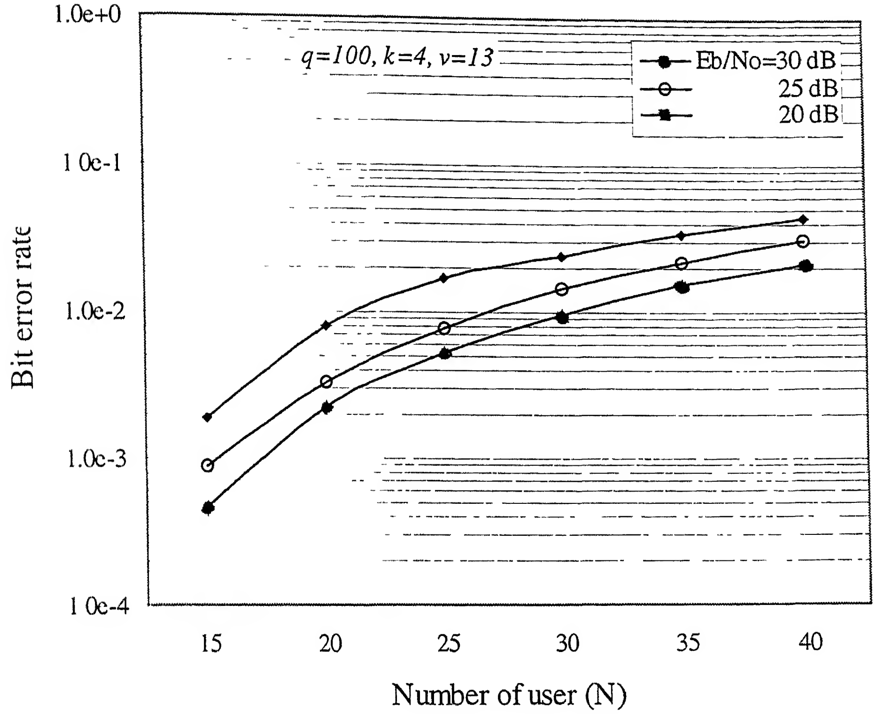


Figure 4.11: Theoretical plot of upper bounds for Steiner system with Eb/No (dB) as parameter for unfaded channel.

4.5.3 Simulation results

The simulation results for non-fading channel are plotted in Fig. 4.12 without using any coding where each point of the curves is the average of 5 different runs. The number of user (N) is 20, 25 and 30. The SNR is varied from 5 dB to 40 dB. The incoming bit rate is 1 kbps. For Steiner system, the number of frequency bins q = 100, diversity L = 3, v = 13 and for MFSK-SFH system, M = 16 and L = 5 other parameters remaining same.

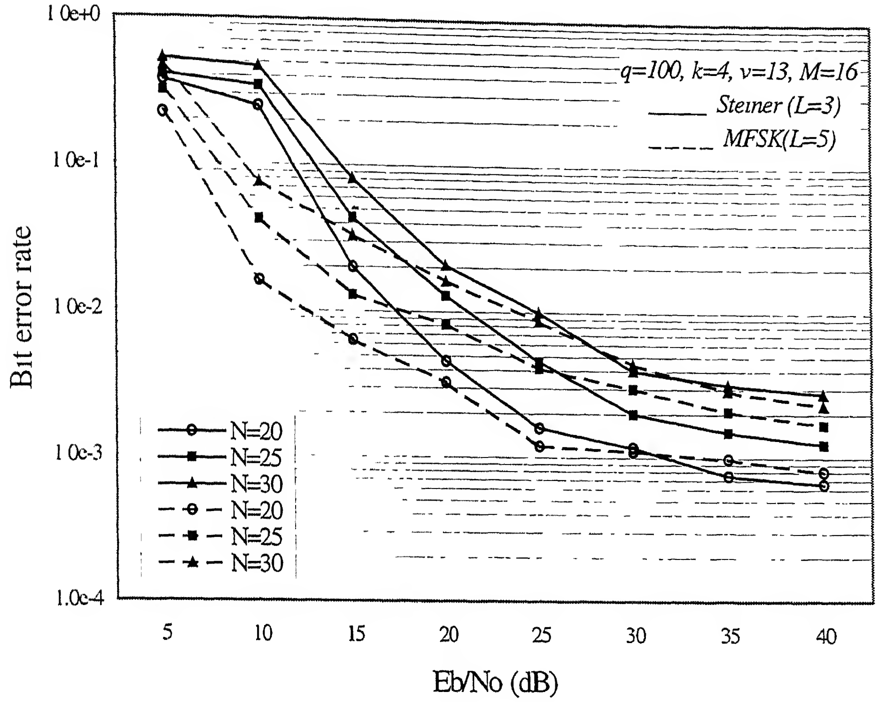


Figure 4.12: Simulation plot of bit error rate vs. Eb/No with N as parameter for unfaded channel.

4.5.4 Comparison between bounds and simulation results

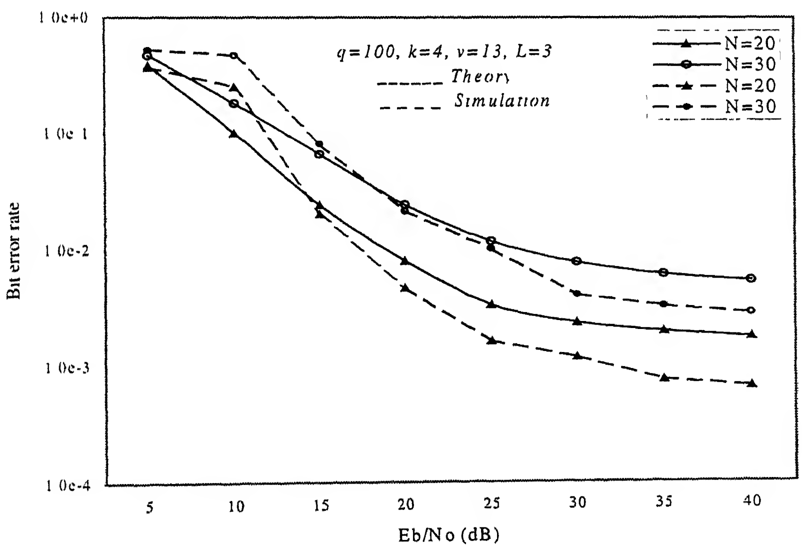


Figure 4.13: Comparison between theoretical upper bounds and simulation results of Steiner system for unfaded channel.

Fig. 4.13 shows a comparison between theoretical and simulation results of the Steiner system for unfaded channel with diversity 3. The numbers of users for each of the cases are 20 and 30 other parameters remaining same.

4.5.5 Performance improvement by coding

The bit error performance in unfaded channel can further be improved by using convolutional coding as shown in Fig. 4.14 where a rate ½ systematic convolutional encoder and Viterbi decoder (using MATLAB functions) are used for the simulation purpose. The curves are plotted for 20 and 30 users keeping other parameters same as above. Figure 4.14 shows that at BER of 0.01 for N = 20 users the coding gain achieved is approximately 2 5 dB.

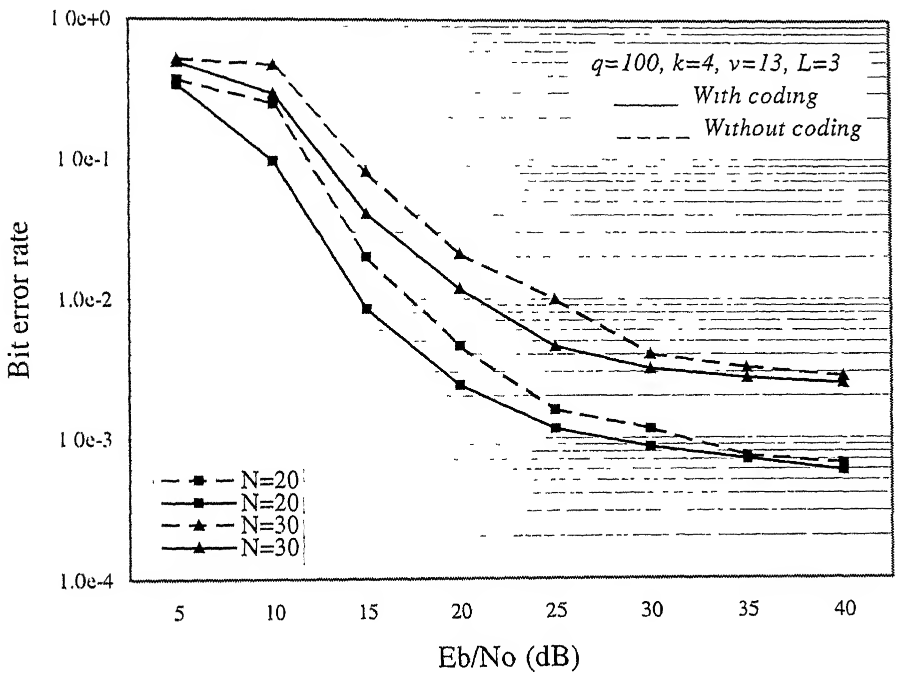


Figure 4.14: Performance improvement of the Steiner system by using rate ½ systematic convolutional coding for unfaded channel.

4.6 Theoretical and Simulation Results for Rayleigh Fading Channel

4.6.1 Choice of optimum diversity

Figure 4.15 shows simulation plot of bit error rate vs. diversity (L) for frequency non-selective Rayleigh fading channel for signal to noise ratio per bit of 25 dB with q = 100, k = 4, L = 3 and v = 13 for Steiner system and with M = 16, L = 5 for MFSK-SFH system. The result is shown for N = 15 20 & 25 number of users. Bit error rate has lower values at the region between L = 1 to 3 for Steiner system while it has the lowest value at L = 5 for MFSK-SFH system. The optimum diversity, therefore, is taken as 3 for Steiner system and 5 for MFSK-SFH system.

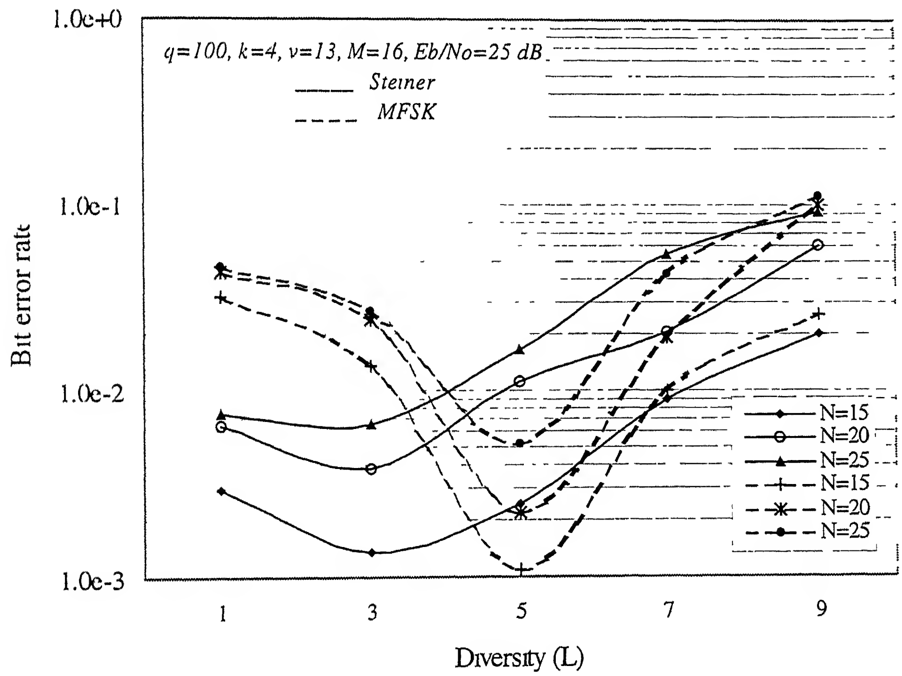


Figure 4.15: Bit error rate vs. diversity plot for different number of user (N) at Eb/No = 25 dB for Rayleigh fading channel.

4.6.2 Theoretical results

Theoretical upper bounds (using equations 3.24(a), 3.24(b), 3.27-3.30, 3.40) for Rayleigh fading channel is plotted in Fig. 4.16 using number of user N as a parameter, other parameters remaining same. The bit error rate sharply decreases with SNR at the

low value of SNR, but at higher values it is practically constant and depends only on the number of users in the system, i.e. the system becomes multiple access interference (MAI) dominated.

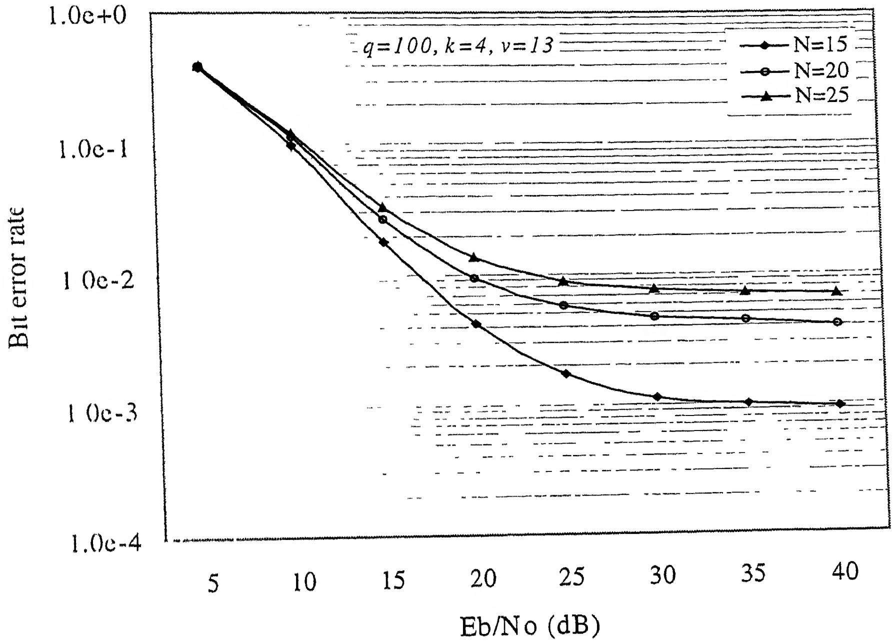


Figure 4.16: Theoretical plot of upper bounds for Steiner system with N as parameter for fading channel.

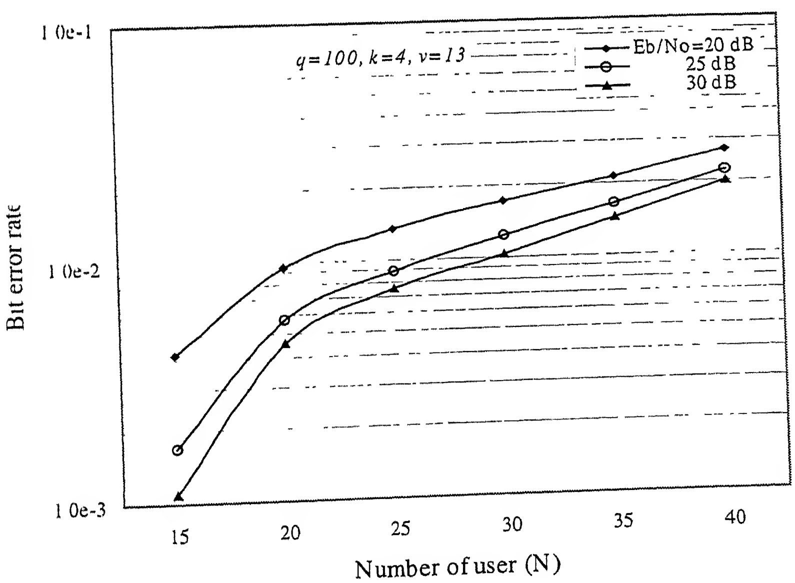


Figure 4.17: Theoretical plot of upper bounds for Steiner system with Eb/No (dB) as parameter for fading channel.

Fig. 4.17 shows theoretical upper bounds (using the same equations as above) for BER vs. number of user (N) plot for fading channel. It is seen from the plot that at BER of 0.01 and Eb/No of 25 dB, more than 25 simultaneous users can be accommodated in the channel of bandwidth 650 kHz.

4.6.3 Simulation results

Simulation results for Rayleigh fading channel are shown in Fig. 4.18 where each point on each curve is an average of 5 simulation data. The number of users N is varied from 15 to 25. If N is below 15, the bit error rate is very small and in order to achieve the accurate BER the required simulation time too much. The other parameters are $q = 100$, $k = 4$, $M = 16$ and $v = 13$. The diversity for Steiner system is 3 and that for MFSK-SFH system is 5. The BER performance for MFSK system is better for lower values of Eb/No because the signal energy is three times that of the Steiner system and becomes comparable to Steiner system at higher SNR. The advantage we get for the System is the better bandwidth efficiency. The ratio of the bandwidth efficiency of the Steiner system to the MFSK system (equations 3.8 & 3.9) is about 2.05.

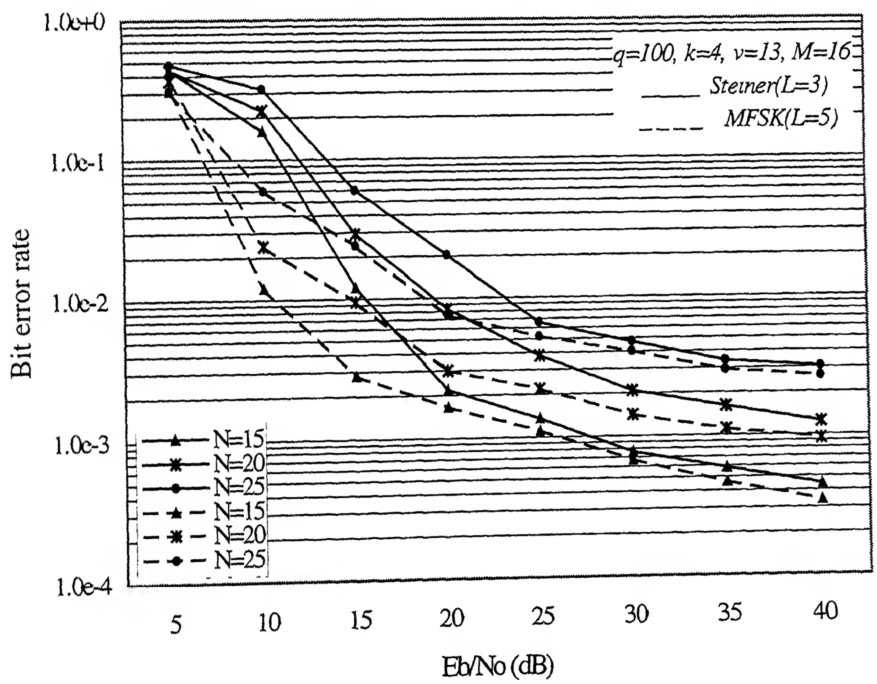


Figure 4.18: Simulation plot of bit error rate vs. Eb/No (dB) with N as parameter for Rayleigh fading channel.

4.6.4 Comparison between bounds and simulation results

A comparative study between theory and simulation results is depicted in Fig. 4.19. The solid curves show theoretical results. The results are shown for N = 20 and 25 users .by varying SNR per bit from 5 dB to 40 dB. Other parameters (q, k, M, v) remain the same.

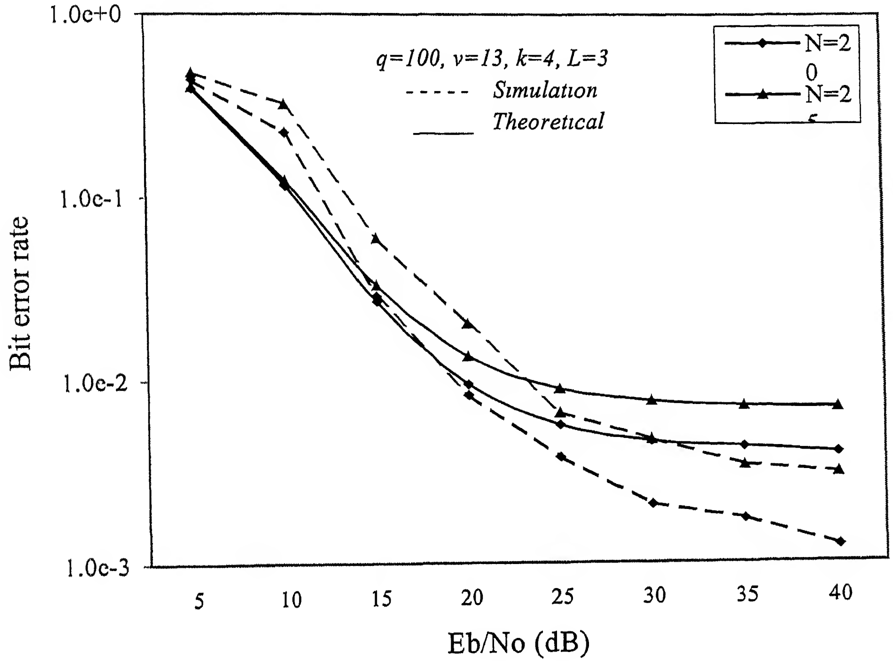


Figure 4.19: Comparison between theoretical upper bounds and simulation results of Steiner system for fading channel.

4.6.5 Performance improvement by coding

The performance of the system in presence of frequency non selective Rayleigh fading channel can further be improved by using coding technique. We have used a rate ½ systematic convolutional encoder and Viterbi decoder (using MATLAB functions) for the simulation purpose. The results with and without using coding are plotted in Fig. 4.20 for N = 20 and 25 number of users. Other simulation parameters are q = 100, k = 4, M = 16 and v = 13. Figure 4.20 gives an estimate of how much coding gain can be achieved with convolutional coding.

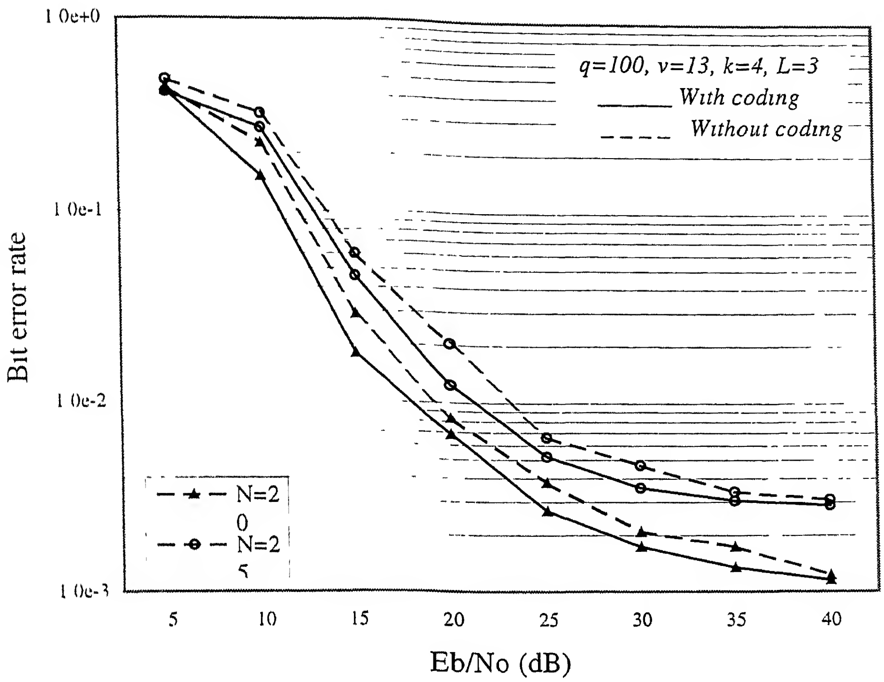
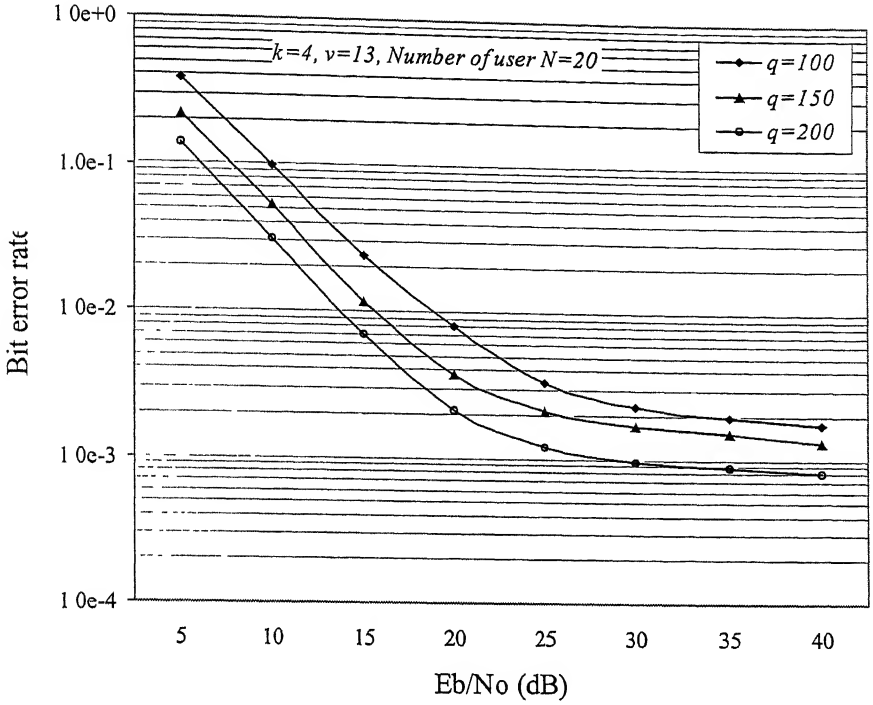


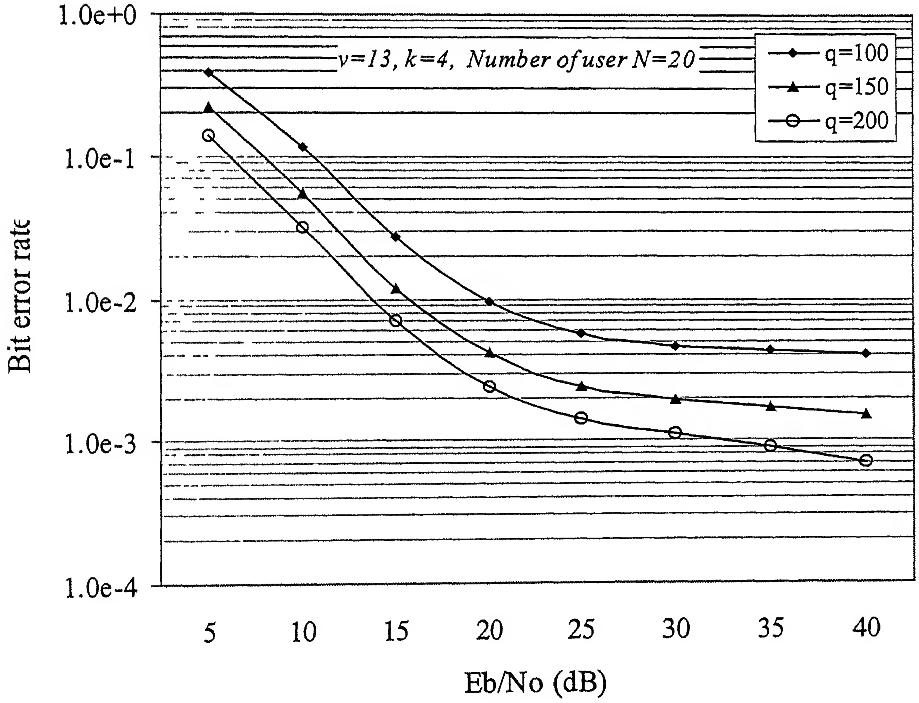
Figure 4 20: Performance improvement of the Steiner system by using rate ½ systematic convolutional coding for Rayleigh fading channel.

4.7 Effect of changing the value of *q*

By changing the value of q (number of frequency bins) better error rate performance can be achieved for a given number of user and signal to noise ratio as shown in Fig. 4.21. Theoretical upper bounds without using any coding are plotted. Fig. 4.21 (a) is shown for unfaded channel with 20 users and Fig. 4.21 (b) is shown for fading channel with $L = 3$ and N = 20 users. Other parameters (v, k, M) are same as above.



(a)



(b)

Figure 4.21: Effect of changing the value of q (theoretical upper bounds) for (a) Non-fading channel, (b) Fading channel.

4.8 Performance of the system with S_4 (v, k) modulation scheme

Up till now we have discussed the scheme with Steiner triple system, i.e. S_3 (v, k). Now we want to study the system with S_4 (v, k) where $v = 25$ (equation 3.5), $b = 50$ (equation 3.3), $r = 8$ (equation 3.4) and $k = 5$. In this system five number of input bits are taken at a time to form an M-ary ($M = 2^k = 2^5$) FSK block and the energy of each such block is divided equally among $w = 4$ Steiner elements. The performance is shown in Fig. 4.22 for fading channel with q = 100 frequency bins and diversity L= 3 for 20 number of users. We have only plotted the theoretical upper bounds without using any coding. The performance is improved due to the increase of effective diversity of the system though the bandwidth efficiency (equation 3.9) is reduced compared to S_3 (v, k) system.

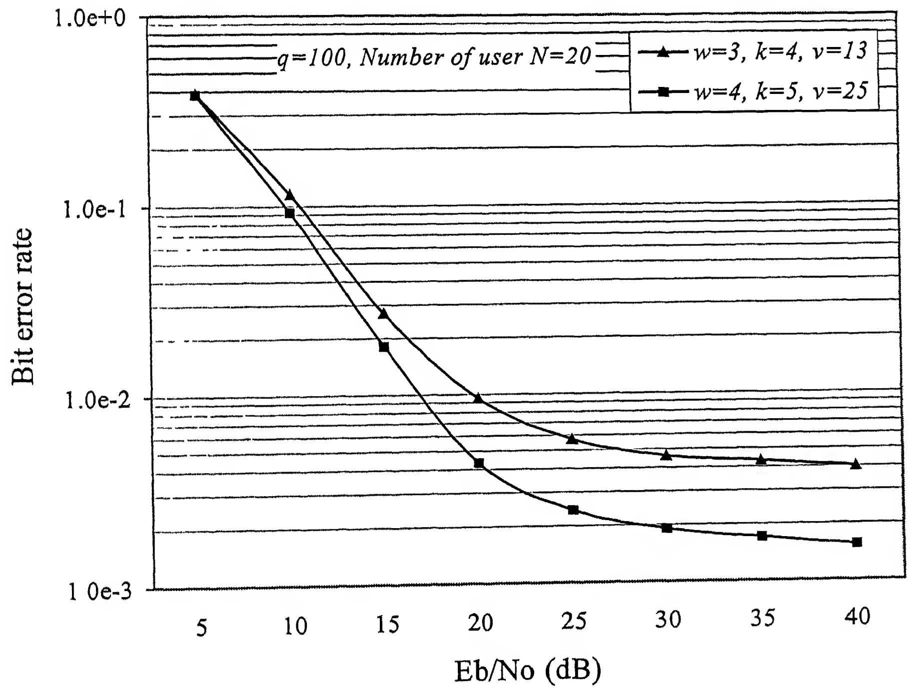


Figure 4.22: Performance with S_4 (v, k) modulation (theoretical upper bounds) for frequency non-selective fading channel with L = 3.

4.9 Performance with Matched Frequency Hopping (MFH) Scheme

Figure 4.23 shows BER performance of the Steiner system with channel matched frequency hopping pattern for 20 number of users. The simulation parameters are same as before. For Eb/No greater than 10 dB a lot of improvement is obtained with such an effective addressing scheme for selective fading channel.

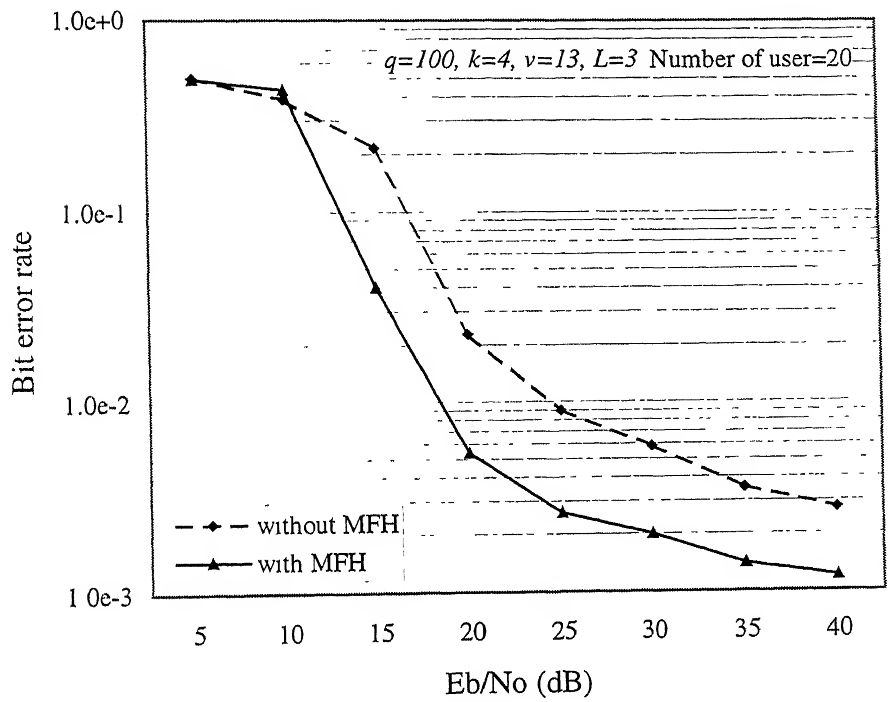


Figure 4.23: Performance of the Steiner system with MFH for frequency selective fading channel.

4.10 Discussions

The system incorporates frequency diversity (simultaneous transmission of L frequency bins for each symbol) and error correcting capability due to the use of the Steiner design. But further improvement of BER is possible with the addition of efficient coding scheme. Though the coding scheme (rate ½ systematic convolutionl coding) used is not optimum for the channel models used here, it justifies the use of coding for the present system.

BER vs. diversity curves are plotted in Fig. 4.9 and in Fig. 4.15. For lower values of code diversity (L) the error rate is high due to channel noise and MAI, but with the increase of diversity the system rapidly recovers most of the losses due to interference from other users. However, beyond the optimum diversity, where the BER minimum is achieved, any further increase in diversity degrades the performance due to interference among the users using a fixed number of available frequency bins (q).

From the pots of BER vs. Eb/No for both unfaded and fading channel, it is seen that the performance of MFSK-SFH system (with diversity 5) is better than that of the Steiner system (with diversity 3) at lower SNR and these two systems are comparable at higher SNR. The only advantage we have got from Steiner system is its bandwidth efficiency. The ratio of the bandwidth efficiency of the Steiner system to the MFSK system (equations 3.8 & 3.9) is about 2.05.

Theoretical and simulation results for unfaded channel gives an estimate of how many simultaneous users can share the given frequency band (1.25 kHz to 651.25 kHz). If the required BER is 10^{-2}, then more than 25 users (Fig. 4.18) can be accommodated at Eb/No of 25 dB for frequency non-selective Rayleigh fading channel using only q = 100 frequency bins.

By increasing the number of frequency bin (q) we can accommodate more number of users for a given BER and signal to noise ratio. If S_4 (v, k) system is used instead of S_3 (v, k) system, we have an improved performance in fading channel. This is because with the increase of w, the minimum hamming distance, (w-1), between the Steiner blocks increases which leads to an increase of effective diversity of the system.

Chapter 5

Conclusion and Scope for Further Work

5.1 Conclusion

A code diversity scheme for a synchronized slow frequency hopped (SFH) spread spectrum multiple access communication system using a bandwidth efficient modulation scheme (Steiner system) has been proposed. The Steiner system performance with optimum code diversity (L) is shown and compared with MFSK-SFH code diversity system for non-fading channel as well as frequency non-selective slow Rayleigh fading channel. Due to the use of frequency diversity (code diversity) and the inherent error correcting capability of Steiner design scheme, the system shows bit error rate performance, which is comparable at higher SNR values to that of the MFSK-SFH system. The advantage obtained with the Steiner system is the higher bandwidth efficiency compared to the MFSK system.

Error correcting code (rate ½ systematic convolutional encoder and Viterbi decoder) can improve the system performance slightly for both non-fading and fading channels. Although coded system requires a larger transmission rate.

The Steiner system with more number of elements per block improves the system performance in Rayleigh fading channels though the bandwidth efficiency of the system is reduced.

The performance of the Steiner system with matched frequency hopping (MFH), an efficient addressing technique for slow frequency selective dispersive channel has also been studied and the BER performance of the system was found to be better for this type of channel.

5.2 Scope for Further Work

(1) The considered system allows each transmitter to send its data over L distinct frequency bins and this number is kept same for all the users. Different L's for different users might be used, in which a high priority transmitter is allowed a larger L.

(3) Although the system is assumed to synchronized, it can be applied to asynchronous systems as well.

(4) The scheme can also be applied to fast frequency hopping (FFH) multiple access systems.

(5) An efficient error correcting coding scheme can be employed with code diversity scheme to further improve the performance of the system.

References

[1] R. L. Pickholz, D. L. Schilling & L. B. Milstein, "Theory of Spread spectrum Communication – A Tutorial," *IEEE Transactions on Communications,* vol. COM-30, No. 5, pp. 855-883, May 1982.

[2] Marvin K. Simon, Jim K. Omura, Robert A. Scholtz & Barry K. Levitt, *Spread Spectrum Communications,* Computer Science Press, Maryland, 1985.

[3] S. M. Schwartz, "Frequency Hopping spread Spectrum (FHSS) vs. Direct Sequence Spread Spectrum (DSSS)," *IEEE 802.11 Wireless Local Area Network Arena*, ver. 3, July 1997.
"http//:www.breezecom.com/TechSupport/fhvsds.htm"

[4] D. Verhulst, M. Mouly & J. Szpirglas, "Slow Frequency Hopping Multiple Access for Digital Cellular Radiotelephone," *IEEE Journal on Selected Areas in Communications,* vol. SAC-2, No. 4, pp. 563-574, July 1984.

[5] R. L. Peterson, R. E. Ziemer & D. E. Borth, *Introduction to Spread Spectrum Communications,* Prentice Hall, New Jersey, 1995.

[6] G. R. Cooper & R. W. Nettleton, "A Spread Spectrum Technique for High-Capacity Mobile Communications," *IEEE Transactions on Vehicular Technology,* vol. VT-27, No. 4, pp. 264-275, November 1978.

[7] D. J. Goodman, P. S. Henry & V. K. Prabhu, "Frequency-Hopped Multilevel FSK for Mobile Radio," *Bell System Technical Journal,* vol. 59, No. 7, pp. 1257-1275, 1980.

[8] U. Timor, "Multistage Decoding of Frequency-Hopped FSK System," *Bell System Technical Journal*, vol. 60, pp. 471-483, April 1981.

[9] B. G. Haskell, "Computer Simulation Results on Frequency Hopped MFSK Mobile Radio-Noiseless Case," *IEEE Transactions on Communications,* vol. COM-29, pp. 125-132, 1982.

[10] J. L. Dornstetter & D. Verhulst, "Cellular Efficiency with Slow Frequency Hopping: Analysis of the Digital SFH900 Mobile System," *IEEE Journal on Selected Areas in Communications,* vol. SAC-5, No. 5, pp 835-848, June 1987.

[11] G. Einarsson, "Address Assignment for a Time-Frequency-Coded Spread Spectrum System," *Bell System Technical Journal,* vol. 59, No. 7, pp. 1241-1255, 1980.

[12] E. L. Titlebaum, "Time frequency Hop Signals – Part I. Coding based upon the Theory of Linear Congruence," *IEEE Transactions on Aerospace Electronic Systems,* vol. AES-17, pp. 490-494, July 1981.

[13] S. V. Maric & E. L. Titlebaum, "Frequency Hop Multiple Access Codes based on the Theory of Cubic Congruence," *IEEE transactions on Aerospace Electronic Systems,* vol. 25, pp. 1035-1039, Nov. 1990.

[14] S. V. Maric & E. L. Titlebaum, "A Class of Frequency Hop Codes with Nearly Ideal Characteristics for Use in Multiple-Access Spread-spectrum Communications and Radar and Sonar Systems," *IEEE Transactions on Communications,* vol. 40, No. 9, pp. September. 1992.

[15] L. D. Wronski, R. Hossain & A. Albicki, "Extended Hyperbolic Congruential Frequency Hop Code: Generation and Bounds for Cross- and Auto-Ambiguity Function," *IEEE Transactions on Communications,* vol. 44, No. 3, pp. 301-305, March 1996.

[16] On-Ching Yue, "Spread Spectrum Mobile Radio, 1977-1982," *IEEE Transactions on Vehicular Technology,* vol. VT-32, No. 1, pp. 98-105, February 1983.

[17] G. E. Atkın & H. P. Corrales, "An Efficient Modulation/Codıng Scheme for MFSK Systems on Bandwidth Constrained Channels," *IEEE Journal on Selected Areas ın Communıcations,* vol. 7, No. 9, pp. 1396-1401, December 1989.

[18] G. E. Atkin & I. F. Blake, "Performance of Multitone FFH/MFSK Systems in the Presence of Jamming," *IEEE Transactıons on Informatıon Theory,* vol. 35, No. 2, pp. 428-435, 1989.

[19] C. T. Ong & C. Leung, "Code Diversity Transmission in a Slow-Frequency-Hopped Spread Spectrum Multiple-Access Communication System," *IEEE Transactions on Communications,* vol. 43, No 12, pp. 2897-2899, December 1995.

[20] W. D. Rummler, "A New Selective Fading Model: Application to Propagation Data," *The Bell System Technical Journal,* vol. 58, No. 5, pp. 1037-1071, May-June 1979.

[21] E. El-Khamy, "Matched Frequency-Hopping (MFH) Signals for Slowly Fadıng Dispersive channels," *IEEE Transactıons on Vehicular technology,* vol. 47, No. 1, pp. 365-369, February 1998.

[22] J. F. Piper, J. G. Proakis, R. R. Reed and J. K. Wolf, "Design of efficient Coding and Modulation for a Rayleigh Fading Channel," *IEEE Transactions on Informatıon Theory*, vol. IT-24, pp. 457-468, July 1978.

[23] J. M. Hall, *Combinatorial Theory,* Second Edition, Wiley-Interscience Publication, 1986.

[24] F. G. McWilliams and N. J. Sloane, *The Theory of Error Correcting Codes,* Amsterdam, The Netherlands: North-Holland, 1977.

[25] M. V. Hegde and W. E. Stark, "On the Error Probability of Coded frequency-Hopped Spread-spectrum Multiple Access systems," *IEEE Transactions on Communications,* vol. 38, no. 5, pp. 571-573, May 1990.

[26] William Feller, *An Introduction to Probability Theory and Its Applications*, vol. 1, Third Edition, New York: Wiley, 1968.

[27] J. G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill Inc, 1995.

[28] C. W. Helstorm, *Probability and Stochastic Processes for Engineers,* New York: Macmillan, 1984.